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THE METRIC PROPERTIES OF A SPACE OF ENTIRE FUNCTIONS OF BOUNDED L-INDEX IN DIRECTION

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The metric properties of a space of entire functions of bounded L-index in direction are investigated. It is shown that the space of entire functions with L-index in direction less than p is of the first category in the topology generated of Iyer's metric.

Key words: *entire function, bounded L-index in direction, bounded value L-distribution in direction, directional derivative.*

K. Ekblaw investigated properties of a space of entire functions of bounded index for one variable in [1]. He proved that in topology generated by metric $d(f,g)=\sup\{|a_0-b_0|, |a_p-b_p|^{1/p}: p\in \mathbb{N}\}$ the entire functions of bounded index, B, are of the first category. Later M. Bordulyak generalized this result for entire functions of several complex variables in [2]. We introduced the entire functions of bounded L-index in direction in [3].

Therefore results of Bordulyak and Ekblaw are generalized for entire functions in $\mathbb{C}^n$ of bounded L-index in direction.

Let $L(z)$, $z\in \mathbb{C}^n$, be a positive continuous function.

Definition 1 *(see[3]). An entire function of $F(z)$, $z\in\mathbb{C}^n$, is called function of bounded L-index in the direction of $\mathbf{b}\in\mathbb{C}^n$, if there exists $m_0\in\mathbb{Z}_+$ such that for $m\in \mathbb{Z}_+$ and every $z\in\mathbb{C}^n$ next inequality is true:*

$$\frac{1}{m!L^m(z)}\left|\frac{\partial^m F(z)}{\partial \mathbf{b}^m}\right| \le \max\left\{\frac{1}{k!L^k(z)}\left|\frac{\partial^k F(z)}{\partial \mathbf{b}^k}\right| : 0\le k\le m_0\right\},$$

where $\frac{\partial^0 F(z)}{\partial \mathbf{b}^0}=F(z)$, $\frac{\partial F(z)}{\partial \mathbf{b}}=\sum_{j=1}^{n}\frac{\partial F(z)}{\partial z_j}b_j$, $\frac{\partial^k F(z)}{\partial \mathbf{b}^k}=\frac{\partial}{\partial \mathbf{b}}\left(\frac{\partial^{k-1}F(z)}{\partial \mathbf{b}^{k-1}}\right), k\ge 2.$

The least such integer m_0 is called the L-index in direction of $F(z)$ and is denoted by $N_{\mathbf{b}}(F,L)$. If such m_0 does not exist then we put $N_{\mathbf{b}}(F,L)=\infty$ and F is said of unbounded L-index in direction. We also denote by $N_{\mathbf{b}}(F,L,z^0)$ as L-index in direction $\mathbf{b}$ of function F in a point z^0 that is the least integer m_0 for which inequality (1) is true at $z=z^0$.

For entire in $\mathbb{C}^n$ functions $F(z)$, $G(z)$ we put

$$d(F,G)=\sup\ \ |F(0)\quad G(0)|,\left|\frac{1}{p!|\mathbf{b}|^{2p}}\frac{\partial^p F(0)}{\partial\mathbf{b}^p}\quad\frac{1}{p!|\mathbf{b}|^{2p}}\frac{\partial^p G(0)}{\partial\mathbf{b}^p}\right|^{1/p}:p\in\mathbb{N}\ ,$$

and a space of entire functions with such metric is denoted $E^n_{\mathbf{b}}$.

Let $B^n_{\mathbf{b}}(L)$ be a set of entire functions in $\mathbb{C}^n$ of bounded L-index in direction and $B^n_{\mathbf{b},\nu}(L)$ be a set of functions with $B^n_{\mathbf{b}}(L)$ such that $N_{\mathbf{b}}(F,L)\leq\nu$. It is clear that $B^n_{\mathbf{b}}(L)=\bigcup_{\nu}B^n_{\mathbf{b},\nu}(L)$.

Besides we denote $a,c\ =\sum_{j=1}^{n}a_j\overline{b_j}$ be a scalar product in $\mathbb{C}^n$, $a,c\in\ \mathbb{C}^n$.

Lemma 1. *For any* $F\in\ E^n_{\mathbf{b}}$, $\nu_0\in\ \mathbb{N}$ *and* $\varepsilon>0$ *there exists* $\delta>0$ *such that if* $G\in\ E^n_{\mathbf{b}}$ *and* $d(F,G)<\delta$ *then* $d\left(\frac{\partial^k F}{\partial\mathbf{b}^k},\frac{\partial^k G}{\partial\mathbf{b}^k}\right)<\varepsilon$ *for* $k=0,1,2,\ldots,\nu_0$.

Proof. Let $F\in\ E^n_{\mathbf{b}}$, $\nu_0\in\ \mathbb{N}$ and $\varepsilon>0$ be given. Let

$$T>\sup\ \ \max\{1,|\mathbf{b}|^{2+2k/p}\ \ \frac{(p+k)!}{p!}^{1/p}:p\in\ \mathbb{N},k=0,1,2,\ldots,\nu_0\ \ .$$

It is straightforward to verify that if $G(z)\in\ E^n_{\mathbf{b}}$ and $d(F,G)<\frac{\varepsilon}{\varepsilon+T}<1$ then

$$d\left(\frac{\partial^k F}{\partial\mathbf{b}^k},\frac{\partial^k G}{\partial\mathbf{b}^k}\right)=\sup\left\{k!|\mathbf{b}|^{2k}\left|\frac{1}{k!|\mathbf{b}|^{2k}}\frac{\partial^k F(0)}{\partial\mathbf{b}^k}-\frac{1}{k!|\mathbf{b}|^{2k}}\frac{\partial^k G(0)}{\partial\mathbf{b}^k}\right|,\left(\frac{(p+k)!|\mathbf{b}|^{2k+2p}}{p!}\times\right.\right.$$

$$\left.\left.\times\left|\frac{1}{(p+k)!|\mathbf{b}|^{2k+2p}}\frac{\partial^{p+k}F(0)}{\partial\mathbf{b}^{p+k}}-\frac{1}{(p+k)!|\mathbf{b}|^{2k+2p}}\frac{\partial^{p+k}G(0)}{\partial\mathbf{b}^{p+k}}\right|\right)^{1/p}:p\in\mathbb{N}\right\}<$$

$$<\sup\left\{k!|\mathbf{b}|^{2k},\left(\frac{(p+k)!|\mathbf{b}|^{2k+2p}}{p!}\right)^{1/p}:p\in\mathbb{N}\right\}\sup\left\{\left|\frac{1}{k!|\mathbf{b}|^{2k}}\frac{\partial^k F(0)}{\partial\mathbf{b}^k}-\frac{1}{k!|\mathbf{b}|^{2k}}\frac{\partial^k G(0)}{\partial\mathbf{b}^k}\right|^{\frac{1}{k}\cdot k},\right.$$

$$\left|\frac{1}{(p+k)!|\mathbf{b}|^{2k+2p}}\frac{\partial^{p+k}F(0)}{\partial\mathbf{b}^{p+k}}\quad\frac{1}{(p+k)!|\mathbf{b}|^{2k+2p}}\frac{\partial^{p+k}G(0)}{\partial\mathbf{b}^{p+k}}\right|^{\frac{1}{p+k}\cdot\frac{p+k}{p}}:p\in\ \mathbb{N}\ <$$

$$<\sup\ \max\{1,|\mathbf{b}|^{2+2k/p}\}\quad\frac{(p+k)!}{p!}^{1/p}:p\in\ \mathbb{N}\ \sup\ \left|\frac{1}{k!|\mathbf{b}|^{2k}}\frac{\partial^k F(0)}{\partial\mathbf{b}^k}\quad\frac{1}{k!|\mathbf{b}|^{2k}}\frac{\partial^k G(0)}{\partial\mathbf{b}^k}\right|^{\frac{1}{k}},$$

$$\left|\frac{1}{(p+k)!|\mathbf{b}|^{2k+2p}}\frac{\partial^{p+k}F(0)}{\partial\mathbf{b}^{p+k}}\quad\frac{1}{(p+k)!|\mathbf{b}|^{2k+2p}}\frac{\partial^{p+k}G(0)}{\partial\mathbf{b}^{p+k}}\right|^{\frac{1}{p+k}}:p\in\mathbb{N}\;<$$

$$<T\sup\;|F(0)\quad G(0)|,\left|\frac{1}{p!|\mathbf{b}|^{2p}}\frac{\partial^{p}F(0)}{\partial\mathbf{b}^{p}}\quad\frac{1}{p!|\mathbf{b}|^{2p}}\frac{\partial^{p}G(0)}{\partial\mathbf{b}^{p}}\right|^{1/p}:p\in\mathbb{N}\;<T\quad\frac{\varepsilon}{T+\varepsilon}<\varepsilon$$

for $k=1,2,...,n$.

Theorem 1 *Let* $F\in E^n_{\mathbf{b}}$, $v\in\mathbb{N}$, $N_{\mathbf{b}}(F,L)>v$. *There exists* $\delta>0$ *such that if* $G\in E^n_{\mathbf{b}}$ *and* $d(F,G)<\delta$ *then* $N_{\mathbf{b}}(G,L)>v$.

Proof. As $N_{\mathbf{b}}(F,L)>v$ then there exists $z^0\in\mathbb{C}^n$ and $v_0>v$ such that $N_{\mathbf{b}}(F,L,z^0)=v_0>v.$ For every $k\le v_0\quad 1$ next inequality is true

$$\frac{1}{k!L^k(z^0)}\left|\frac{\partial^k F(z^0)}{\partial\mathbf{b}^k}\right|<\frac{1}{v_0!L^{v_0}(z^0)}\left|\frac{\partial^{v_0} F(z^0)}{\partial\mathbf{b}^{v_0}}\right|.$$

This inequality is strict because v_0 is least integer for constrict inequality at point $z^0.$ Then there exists $\delta^*>0$ such that

$$\frac{1}{k!L^k(z^0)}\left|\frac{\partial^k F(z^0)}{\partial\mathbf{b}^k}\right|+\delta^*<\frac{1}{v_0!L^{v_0}(z^0)}\left|\frac{\partial^{v_0} F(z^0)}{\partial\mathbf{b}^{v_0}}\right|.$$

It is obviously that

$$|a\quad c|+|b\quad d|\ge\;(a\quad c)+(b\quad d).$$

Then $a\quad b\ge c\quad d\quad |a\quad c|\quad |b\quad d|$. We apply this inequality and we have

$$\frac{1}{v_0!L^{v_0}(z^0)}\left|\frac{\partial^{v_0}G(z^0)}{\partial\mathbf{b}^{v_0}}\right|-\frac{1}{k!L^k(z^0)}\left|\frac{\partial^{k}G(z^0)}{\partial\mathbf{b}^{k}}\right|\ge\frac{1}{v_0!L^{v_0}(z^0)}\left|\frac{\partial^{v_0}F(z^0)}{\partial\mathbf{b}^{v_0}}\right|-\frac{1}{k!L^k(z^0)}\left|\frac{\partial^{k}F(z^0)}{\partial\mathbf{b}^{k}}\right|-$$

$$-\frac{1}{v_0!L^{v_0}(z^0)}\left\|\frac{\partial^{v_0}G(z^0)}{\partial\mathbf{b}^{v_0}}\right|-\left|\frac{\partial^{v_0}F(z^0)}{\partial\mathbf{b}^{v_0}}\right\|-\frac{1}{k!L^k(z^0)}\left\|\frac{\partial^{k}G(z^0)}{\partial\mathbf{b}^{k}}\right|-\left|\frac{\partial^{k}F(z^0)}{\partial\mathbf{b}^{k}}\right\|. \qquad (3)$$

Using idea of proof Taylor's formula we can prove that

$$F(z)=\sum_{p=0}^{\infty}\frac{1}{p!|\mathbf{b}|^{2p}}\frac{\partial^p F(0)}{\partial\mathbf{b}^p}\quad z,\mathbf{b}\quad^p$$

and

$$G(z)=\sum_{p=0}^{\infty}\frac{1}{p!|\mathbf{b}|^{2p}}\frac{\partial^p G(0)}{\partial\mathbf{b}^p}\quad z,\mathbf{b}\quad^p.$$

Hence clearly

$$\left\|\frac{\partial^k G(z^0)}{\partial\mathbf{b}^k}\right|-\left|\frac{\partial^k F(z^0)}{\partial\mathbf{b}^k}\right\|\le d\left(\frac{\partial^k F}{\partial\mathbf{b}^k},\frac{\partial^k G}{\partial\mathbf{b}^k}\right)+\sum_{j=1}^{\infty}\left(d\left(\frac{\partial^k F}{\partial\mathbf{b}^k},\frac{\partial^k G}{\partial\mathbf{b}^k}\right)\right)^j\;|\langle z^0,\mathbf{b}\rangle|^j$$

By Lemma 1 we can choose a number δ such that if $d(F,G)<\delta$ then $d\left(\frac{\partial^k F}{\partial \mathbf{b}^k},\frac{\partial^k G}{\partial \mathbf{b}^k}\right)<\varepsilon<1$ and $d\left(\frac{\partial^k F}{\partial \mathbf{b}^k},\frac{\partial^k G}{\partial \mathbf{b}^k}\right| z^0,\mathbf{b} |<\varepsilon<1$ for all $k\le v_0$. Therefore, for all $k\le v_0$

$$\left\|\left|\frac{\partial^k G(z^0)}{\partial \mathbf{b}^k}\right| - \left|\frac{\partial^k F(z^0)}{\partial \mathbf{b}^k}\right|\right\| \le \varepsilon+\frac{\varepsilon}{1-\varepsilon}=\frac{\varepsilon(2-\varepsilon)}{1-\varepsilon}$$

and from (2) and (3) for all $k\le v_0-1$ we have

$$\frac{1}{\nu_0!L^{\nu_0}(z^0)}\left|\frac{\partial^{\nu_0}G(z^0)}{\partial \mathbf{b}^{\nu_0}}\right|-\frac{1}{k!L^k(z^0)}\left|\frac{\partial^k G(z^0)}{\partial \mathbf{b}^k}\right|\ge\delta^*-\frac{\varepsilon(2-\varepsilon)}{1-\varepsilon}\left(\frac{1}{\nu_0!L^{\nu_0}(z^0)}+\frac{1}{k!L^k(z^0)}\right)$$

whence, in view of arbitrary of ε, it follows

$$\frac{1}{\nu_0!L^{\nu_0}(z^0)}\left|\frac{\partial^{\nu_0}G(z^0)}{\partial \mathbf{b}^{\nu_0}}\right|-\frac{1}{k!L^k(z^0)}\left|\frac{\partial^k G(z^0)}{\partial \mathbf{b}^k}\right|>\frac{\delta^*}{2}$$

for all $k\le v_0-1$ that is $\infty\ge N_{\mathbf{b}}(L,G)\ge N_{\mathbf{b}}(L,G,z^0)\ge v_0>v$. Theorem 1 is proved.

Remark 1 *The condition $N_{\mathbf{b}}(F,L)>v$ in Theorem 1 is equivalent that $F\in E^n_{\mathbf{b}}\setminus B^n_{\mathbf{b},v}(L)$. Therefore we can reformulate the Theorem 1.*

Theorem *Let $F\in E^n_{\mathbf{b}}$, $v\in \mathbb{N}$, $F\in E^n_{\mathbf{b}}\setminus B^n_{\mathbf{b},v}(L)$. There exists $\delta>0$ such that if $G\in E^n_{\mathbf{b}}$ and $d(F,G)<\delta$ then $G\in E^n_{\mathbf{b}}\setminus B^n_{\mathbf{b},v}(L)$.*

Corollary 1 *The set $B^n_{\mathbf{b},v}(L)$ is closed in E^n.*

Lemma 2 *If $P(z)$ is a polynomial of degree p then $F(z)=\exp\langle z,a\rangle+P(z)$ is of L-index in direction $\mathbf{b}\in\mathbb{C}^n$ less or equal $p+1$, where $a\in\mathbb{C}^n$, $L(z)=\max\{1,|\langle\mathbf{b},a\rangle|\}$, $\langle\mathbf{b},a\rangle\ne 0$. If $\langle\mathbf{b},a\rangle\ne 0$ then $F(z)$ is index in direction $\mathbf{b}\in\mathbb{C}^n$ of 0.*

Proof. Let $k>p+1$. Thus

$$\frac{1}{k!L^k(z)}\left|\frac{\partial^k F(z)}{\partial^k \mathbf{b}}\right|=\frac{|\exp\langle z,a\rangle|}{k!L^k(z)}|\langle\mathbf{b},a\rangle|^k\le\frac{|\exp\langle z,a\rangle|}{k!}<\frac{|\exp\langle z,a\rangle|}{(p+1)!L^{p+1}(z)}|\langle\mathbf{b},a\rangle|^{p+1}=$$

$$=\frac{1}{(p+1)!L^{p+1}(z)}\left|\frac{\partial^{p+1}F(z)}{\partial^{p+1}\mathbf{b}}\right|$$

and hence $F(z)$ is of L-index in direction $\mathbf{b}\in\mathbb{C}^n$ less or equal $p+1$. If $\langle\mathbf{b},a\rangle\ne 0$ then for all $k>1$ we have

$$\frac{1}{k!L^k(z)}\left|\frac{\partial^k F(z)}{\partial^k \mathbf{b}}\right|=\frac{|\exp\langle z,a\rangle|}{k!L^k(z)}|\langle\mathbf{b},a\rangle|^k\le\frac{|\exp\langle z,a\rangle|}{k!}=0.$$

Therefore $F(z)$ is index in direction $\mathbf{b}\in \mathbb{C}^n$ of 0.

We denote

$$\lambda_1^{\mathbf{b}}(z,t_0,\eta)=\inf\left\{\frac{L(z+t\mathbf{b})}{L(z+t_0\mathbf{b})}:|t-t_0|\le\frac{\eta}{L(z+t_0\mathbf{b})}\right\},$$

$$\lambda_2^{\mathbf{b}}(z,t_0,\eta)=\sup\left\{\frac{L(z+t\mathbf{b})}{L(z+t_0\mathbf{b})}:|t-t_0|\le\frac{\eta}{L(z+t_0\mathbf{b})}\right\},$$

$$\lambda_1^{\mathbf{b}}(z,\eta)=\inf\{\lambda_1^{\mathbf{b}}(z,t_0,\eta):t_0\in \mathbb{C}\},\lambda_2^{\mathbf{b}}(z,\eta)=\inf\{\lambda_2^{\mathbf{b}}(z,t_0,\eta):t_0\in \mathbb{C}\},$$

$$\lambda_1^{\mathbf{b}}(\eta)=\inf\{\lambda_1^{\mathbf{b}}(z,\eta):z\in \mathbb{C}^n\},\ \lambda_2^{\mathbf{b}}(\eta)=\inf\{\lambda_2^{\mathbf{b}}(z,\eta):z\in \mathbb{C}^n\}.$$

A class of functions L, which satisfy the condition $0<\lambda_1^{\mathbf{b}}(\eta)\le\lambda_2^{\mathbf{b}}(\eta)<+\infty$ for all $\eta\le 0$, we denote by $Q_{\mathbf{b}}^n$. We need the following lemma.

Lemma 3 *If* $L\in Q_{\mathbf{b}}^n$ *and an entire transcendental function* $F(z)$ *is of bounded* L*-index in direction* $b\in \mathbb{C}^n$, *then for all* $z^0\in \mathbb{C}^n$

$$\ln M(r,F,z^0)=O\left(\int_0^r L(z^0+t\mathbf{b})dt\right),\ r\to+\infty,$$

where $M(r,F,z^0)=\max\{|F(z^0+t\mathbf{b})|:|t|=r\}$.

Proof. The proof follows from the same lemma for the case of functions of one variable (see Theorem 3.3 on page 71 in [4]) and the fact that $F(z^0+t\mathbf{b})$ is a function of one variable $t\in \mathbb{C}$ if z^0 is fixed.

Theorem 2 *If* $L\in Q_{\mathbf{b}}^n$ *then for every* $v\in \mathbb{Z}_+$ *the set* $B_{\mathbf{b},v}^n(L)$ *is nowhere dense in* $B_{\mathbf{b}}^n(L)$ *and thus* $B_{\mathbf{b}}^n(L)$ *is of the first category. The sets* $E_{\mathbf{b}}^n\setminus B_{\mathbf{b},v}^n(L)$ *and* $E_{\mathbf{b}}^n\setminus B_{\mathbf{b}}^n(L)$ *is dense in* $E_{\mathbf{b}}^n$

Proof. Let $F(z)=\sum_{p=0}^{\infty}F_p\langle z,\mathbf{b}\rangle^p$ be an entire function such that for all $z^0\in \mathbb{C}^n$

$$\frac{\ln M(r,F,z^0)}{\int_0^r L(z^0+t\mathbf{b})dt}\to+\infty,\ r\to+\infty,$$

where $M(r,F,z^0)=\max\{|F(z^0+t\mathbf{b})|:|t|=r\}$. Then by Lemma 3 F is of unbounded L-index in direction $b\in\mathbb{C}^n$. Let $f(z)=\sum_{p=0}^{\infty}f_p\ z,\mathbf{b}\ ^p$ be an entire function of bounded L-index $N_{\mathbf{b}}(F,L)$ in direction *b*. We denote $f_j^*(z)=\sum_{p=0}^{j}\ z,\mathbf{b}\ ^p+\sum_{p=j+1}^{\infty}F_p\ z,\mathbf{b}\ ^p$ and $f_{j,m}(z)=\sum_{p=0}^{j}\ z,\mathbf{b}\ ^p+\sum_{p=j+1}^{m}F_p\ z,\mathbf{b}\ ^p$, where $m>j$.

Then f_j^* is of unbounded L-index in direction **b** for any j i. e. $N_{\mathbf{b}}(L, f_j^*) > N,\ j > 0.$ It is easy see that $d(f.f_j^*) \to 0,\ d(f_j^*, f_{j,m}^*) \to 0$ and $d(f, f_{j,m}^*) \to 0$ as $j \to \infty$. By Theorem 1 $N_{\mathbf{b}}(l, f_{j,m}^*) > N$ for sufficiently large m. On other hand $N(l; f_{j,m}^*) \le m.$ Thus, $f_{j,m}^* \in B_{\mathbf{b}}^n(L) \setminus B_{\mathbf{b},v}^n(L)$ that is $B_{\mathbf{b},v}^n(L)$ is nowhere dense in $B_{\mathbf{b}}^n(L).$

If f is any function from $B^n\mathbf{b}(L),$ we choose f_j^* as above. Then $f_j^* \in E_{\mathbf{b}}^n \setminus B_{\mathbf{b}}^n(L)$ and $d(f, f_j^*) \to 0$ as $j \to \infty.$ Thus, the sets $E_{\mathbf{b}}^n \setminus B_{\mathbf{b}}^n(L)$ is dense in $E.$

Finally, let $f \in E_{\mathbf{b}}^n \setminus (B_{\mathbf{b}}^n(L) \setminus B_{\mathbf{b},v}^n(L))$ i. e. either f is unbounded L-index in direction **b** or $N_{\mathbf{b}}(f, L) \le N.$ We will show that in both cases f is limiting for some functions f_j^* with $N < N_{\mathbf{b}}(l; f_j) < +\infty.$ In the first case we choose

$$f_j^*(z) = \sum_{p=0}^{j} f_n \langle z, \mathbf{b} \rangle^p.$$

Then $d(f, f_j^*) \to 0$ as $j \to +\infty$ and by Theorem 1 $N_{\mathbf{b}}(L, f_j^*) \le N$ for large j. In the second case for $j \in B_{\mathbf{b},v}^n(L)$ we choose, as above, $f_{j,m}^* \in B_{\mathbf{b}}^n(L) \setminus B_{\mathbf{b},v}^n(L).$ Theorem 2 is proved.

References

1. Ekblaw K.A. The functions of bounded index as a subspace of a space of entire functions / K.A.Ekblaw // Pacific J. Math. – 1971. – V.37. – P. 353-355.
2. Bordulyak M.T. The space of entire in $\mathbb{C}^n$ functions of bounded L-index / M.T.Bordulyak // Math. Stud. – 1995. – V.4. – P. 53-58 (in Ukrainian).
3. Bandura A.I. Entire functions of bounded L-index in direction / A.I.Bandura, O.B.Skaskiv // Math. Stud. – 2007. – **27**, N.1. – P. 30-52 (in Ukrainian).
4. Sheremeta M. Analytic functions of bounded index / M.Sheremeta // Sheremeta Myroslav. – Lviv: VNTL Publishers, 1999. – 141 p.

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МЕТРИЧНІ ВЛАСТИВОСТІ ПРОСТОРУ ЦІЛИХ ФУНКЦІЙ ОБМЕЖЕНОГО L-ІНДЕКСУ ЗА НАПРЯМОМ

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Досліджено метричні властивості простору цілих у $\mathbb{C}^n$ *функцій обмеженого* L*-індексу за напрямом. Доведено, що простір цілих функцій,* L*-індекс за напрямом яких не перевищує* p*, є простором першої категорії у топології, породженій метрикою Ієра.*

Ключові слова: *ціла функція, обмежений* L*-індекс за напрямом, метрика Ієра, простір першої категорії, похідна за напрямом.*