МАТЕМАТИКА ТА МЕХАНІКА

Математичне моделювання та обчислювальні методи

УДК 338.5
MSC 2020: 91B41; 91B06, 91B24
DOI: 10.31471/2304-7399-2026-22(83)-112-124

MODEL OF NON-LINEAR PRICING WITH LOWER CONSTRAINTS ON BUYERS' UTILITIES

I.V. Nykyforchyn*
Vasyl Stefanyk Carpathian National University;
76018, 57 Shevchenka street, Ivano-Frankivsk, Ukraine
e-mail: iryna.nykyforchyn@cnu.edu.ua

We extend the model of non-linear pricing by Maskin and Riley, which is classical in contract theory. Buyers' utilities are constrained from below, and it is shown that the statement that only local downward constraints are essential, and all of them are binding, fails in this case. We prove that in our model it is necessary and sufficient to consider also the comparisons of each type with the next type, and strict inequalities in the comparisons of a type with all neighbors are possible only if this type receives the minimal acceptable utility. We propose methods for solving this problem for two types of buyers and conditions for the offers for the both or for one of the types to be effective (to provide maximum of social surplus).

Key words: *price discrimination, monopolistic seller, local constraints, expected utility.*

*Corresponding author

1. Introduction

In this paper we extend the classical model of non-linear pricing by [7], for discrete sets of buyers. Although a long period of time passed since the mentioned work came out, its results are still widely used in analysis of contracts with hidden information. E.g., [2], rely on them in studies of robust monopolistic pricing under ambiguity. In [4], this paper was called “a classical reference on price discrimination”. Nowadays its ideas are adapted, e.g., to develop optimal offers for users of communication networks ([8]). Undoubtedly, this is due to successful mathematical model and natural not very restrictive assumptions, which are satisfied in most of the related economic circumstances. It is assumed, in particular, that the types of buyers can be ranked and parameterized with real numbers so that greater value corresponds to greater demand in considered product. It is known ([6]) that passing even to two-dimensional parameters significantly complicates the analysis and narrows the obtained results. We however are going in this and subsequent papers to study consequences of dropping two assumptions of the model, namely about willingness of a buyer to buy even if his utility is zero, and about obligation of a seller to propose an acceptable offer for each type of buyer.

2. General Principles of the Model

Following the style and, wherever possible, notation of [7], we assume that a monopolist produces a single product at a constant marginal cost c. A buyer of type i has preferences determined by the utility function

$$U = \int_0^q p(x, v_i)\,dx - T,$$

where q is the quantity of units purchased, T is total spending on these units, v_i is a parameter that describes buyer’s type. The seller does not observe v_i but knows a priori distribution $F(v)$ of buyers’ preferences. For each v considered we assume that the demand price $p(q,v)$ strictly decreases in q, and there is (necessarily unique) $q_v^c > 0$ such that $p(q_v^c, v) = c$. The seller’s profit (his utility) equals then

$$R = T - cq = \int_0^q p(x, v_i)\,dx - cq - U = \int_0^q (p(x, v_i) - c)\,dx - U = N(q,v) - U,$$

where $N(q,v)$ is the social surplus generated by the sale. It is easy to see then that the social surplus increases for $0 \le q \le q_v^c$, attains its maximum for $q = q_v^c$, and is decreasing for $q \ge q_v^c$. It is also assumed that $p(q,v)$

strictly increases in v, which implies that q_v^c is strictly increasing in v as well. Indifference curves for different buyers' types and different utility levels will then be as depicted in Fig. 1.

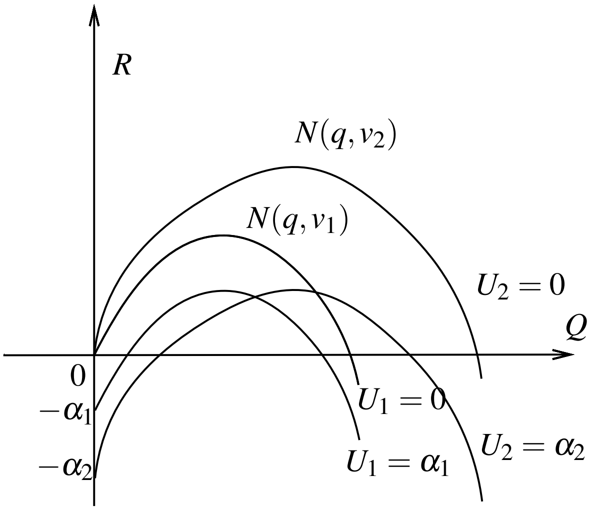

Figure 1: Indifference curves

A selling procedure is a schedule of pairs $(\hat{q}_s, \hat{T}_s)_{s\in S}$, which the seller offers to the buyers. If a buyer chooses s, then he receives q_s units of the product and pays a tariff T_s. Since tariff and seller's utility are linked via the equality $T = R + cq$, it is more convenient for our needs to specify a selling procedure by a schedule $(\hat{q}_s, \hat{R}_s)_{s\in S}$ of the equivalent pairs of product quantity and seller's profit.

From the buyer's viewpoint, if the latter's type is determined by the parameter v, a pair (q^*, R^*) precedes (i.e., yields to) a pair (q^{**}, R^{**}) if $U^* = N(q^*, v) - R^* \leq U^{**} = N(q^{**}, v) - R^{**}$. If the strict inequality holds, then one pair strictly precedes another.

The mentioned model has undergone many modifications to reflect different aspects of buyers' and seller's behavior and interaction. E.g., [5], investigated the influence of buyer's self-control on his willingness to buy.

Our extension to the model by Maskin & Riley is the following: a buyer of type i is not always ready to accept an offer even if his utility is close to zero, but rejects it if his utility is less that some minimal level $b_i = b(v_i) \geq 0$. It means that a buyer of type i never accepts an offer (q^*, R^*) if the respective point is below the indifference curve $N(q, v_i) - R = b_i$.

A seller can respectively prepare for some buyer's type i one optimal (w.r.t. his preference) pair $(\hat{q}_i, \hat{R}_i)$ and prepare no such pair for another type (e.g., if the rate of this type is too small). Obviously there is no sense to offer more than one variant for each buyer's type. We investigate properties of the optimal procedure, deferring consideration of probable ignoring some buyers' types to a subsequent publication.

3. Analysis of the Model

We consider first a case of a finite number of buyer's types, which are specified by parameters $v_1 < v_2 < \cdots < v_{n-1} < v_n$, and their rates are resp. $\pi_i > 0$, $\pi_1 + \pi_2 + \cdots + \pi_{n-1} + \pi_n = 1$, and *each* of them is assigned an optimal (comparing to the others) acceptable offer $(\hat{q}_i, \hat{R}_i)$. Then the following conditions must be valid:

$$N(\hat{q}_i, v_i) - \hat{R}_i \geq b_i, \quad 1 \leq i \leq n, \qquad (IR_i)$$

and

$$N(\hat{q}_i, v_i) - \hat{R}_i \geq N(\hat{q}_j, v_i) - \hat{R}_j, \quad 1 \leq i, j \leq n,\ i \neq j. \qquad (IC_{i,j})$$

The first condition ensures that a buyer's utility is not less than some lower bound. The second condition is called the self-selection constraint in contract theory. It means that there is no reason for a buyer of type i to choose an offer prepared for a buyer of another type j.

The seller then maximizes the expected utility $ER = \sum_{i=1}^{n} \pi_i \hat{R}_i$. Since the set of acceptable sets $(\hat{q}_i, \hat{R}_i)_{1 \leq i \leq n}$ is bounded and specified by the conditions (IR_i) and $(IC_{i,j})$, hence is closed, there exists an optimal (for the seller) selling procedure. In the sequel, if otherwise is not said explicitly, we assume that $(\hat{q}_i, \hat{R}_i)_{1 \leq i \leq n}$ is optimal.

The analysis carried out in [7], without restrictions on buyers' utilities showed that it is always disadvantageous for a buyer to choose an offer for a higher type, and it can be profitable only to choose an offer for the closest lower type, hence, to find an optimal selling procedure, it is sufficient to consider only the constraints $(IC_{i,i-1})$, which are called local downward constraints in the paper.

Moreover, the mentioned local downward constraints are binding, which simplifies the optimization problem for an optimal selling procedure. Unfortunately, if lower constraints on buyers' utilities are introduced, things get more complicated.

Proposition 3.1. *If for a selling procedure* $(\hat{q}_i, \hat{R}_i)_{1 \leq i \leq n}$ *for all* $i = 1, 2, \ldots, n$ *the conditions*

$$N(\hat{q}_i, v_i) - \hat{R}_i \geq b_i, \quad 1 \leq i \leq n, \qquad (IR_i)$$

$$N(\hat{q}_i, v_i) - \hat{R}_i \geq N(\hat{q}_{i-1}, v_i) - \hat{R}_{i-1} \text{ or } i = 1, \tag{DW_i}$$

$$N(\hat{q}_i, v_i) - \hat{R}_i \geq N(\hat{q}_{i+1}, v_i) - \hat{R}_{i+1} \text{ or } i = n, \tag{UW_i}$$

are satisfied, then each pair $(\hat{q}_i, \hat{R}_i)$ *is acceptable and optimal for the respective buyer's type.*

This means that, additionally to local downward constraints, it is necessary to verify also local upward constraints, i.e., to compare each proposition with the closest neighbors on the both sides. Note that the question of sufficiency of local constraints in rather general settings was studied by [3].

Proof. Observe that, for $i < j < k$ the inequalities

$$N(\hat{q}_i, v_i) - \hat{R}_i \geq N(\hat{q}_j, v_i) - \hat{R}_j$$

and

$$N(\hat{q}_j, v_j) - \hat{R}_j \geq N(\hat{q}_k, v_j) - \hat{R}_k$$

imply

$$N(\hat{q}_i, v_i) - \hat{R}_i > N(\hat{q}_k, v_i) - \hat{R}_k.$$

Indeed,

$$(N(\hat{q}_k, v_j) - N(\hat{q}_j, v_j)) - (N(\hat{q}_k, v_i) - N(\hat{q}_j, v_i)) =$$
$$= \int_{\hat{q}_j}^{\hat{q}_k} p(x, v_j)\,dx - \int_{\hat{q}_j}^{\hat{q}_k} p(x, v_i)\,dx = \int_{\hat{q}_j}^{\hat{q}_k} (p(x, v_j) - p(x, v_i))\,dx > 0.$$

Hence

$$N(\hat{q}_k, v_i) - \hat{R}_k < N(\hat{q}_k, v_j) - N(\hat{q}_j, v_j) + N(\hat{q}_j, v_i) - \hat{R}_k \leq$$
$$\leq (R_k - R_j) + N(\hat{q}_j, v_i) - \hat{R}_k = N(\hat{q}_j, v_i) - \hat{R}_j \leq N(\hat{q}_i, v_i) - \hat{R}_i.$$

This means that if it is unprofitable for the i-th type to choose the offer for the j-th one, and it is unprofitable for the j-th type to choose the offer for the k-th one, then its is disadvantageous for the i-th type to choose the offer prepared for the k-th type for $i < j < k$. Therefore it is unprofitable for any type to choose an offer for a higher one (analogously for a lower one). We obtain the optimality of each offer, and the acceptability is guaranteed by the conditions (IR_i). □

Proposition 3.2. *If a selling procedure* $(\hat{q}_i,\hat{R}_i)_{1\le i\le n}$ *is optimal for the seller, in particular, if all pairs* $(\hat{q}_i,\hat{R}_i)$ *are acceptable and optimal for the respective buyers' types, then:*

(a) for all $i=1,2,\ldots,n$ *conditions* (IR_i), (DW_i), (UW_i) *are valid.*

(b) the stronger properties

$$N(\hat{q}_i,v_i)-\hat{R}_i > N(\hat{q}_{i-1},v_i)-\hat{R}_{i-1} \text{ or } i=1$$

and

$$N(\hat{q}_i,v_i)-\hat{R}_i > N(\hat{q}_{i+1},v_i)-\hat{R}_{i+1} \text{ or } i=n$$

can hold simultaneously only if

$$b_i = N(\hat{q}_i,v_i)-\hat{R}_i.$$

(c) the sequence $\hat{q}_i$ *is non-decreasing.*

Proof. (a) It is clear that the inequality $N(\hat{q}_i,v_i)-\hat{R}_i < N(\hat{q}_{i-1},v_i)-\hat{R}_{i-1}$ is impossible because otherwise a buyer of type i could increase his utility by choosing the $(i-1)$-th type's offer. Analogously we exclude $N(\hat{q}_i,v_i)-\hat{R}_i < N(\hat{q}_{i+1},v_i)-\hat{R}_{i+1}$. The inequality $b_i > N(\hat{q}_i,v_i)-\hat{R}_i$ contradicts our assumption on b_i. Thus (IR_i), (DW_i), (UW_i) are valid.

(b) If the two strict inequalities from the thesis were valid together with $b_i < N(\hat{q}_i,v_i)-\hat{R}_i$, or one of them was valid and the other was senseless because the type i is the first or the last one, then the seller could increase his profit $\hat{R}_i$ by a sufficiently small $\delta>0$ so that all the respective conditions remain valid, and therefore increase his expected utility, which contradicts the optimality.

(c) If $\hat{q}_{i+1}<\hat{q}_i$, then (DW_{i+1}) and (UW_i) cannot hold simultaneously. □

This statement means that the point $(\hat{q}_{i-1},\hat{R}_{i-1})$ or the point $(\hat{q}_{i+1},\hat{R}_{i+1})$ cannot be below the indifference curve for the type i drawn through $(\hat{q}_i,\hat{R}_i)$, and can be higher than the curve only if the latter curve corresponds to the minimal acceptable utility b_i.

Remark 3.1. *Unfortunately, under lower constraints on buyers' utilities, unlike the model in [7], the value* $\hat{q}_i$ *is not always less or equal to the optimal point* $q^c_{v_i}$ *for social surplus* $N(q,v_i)$*, e.g., the placement as in Fig. 2 is possible.*

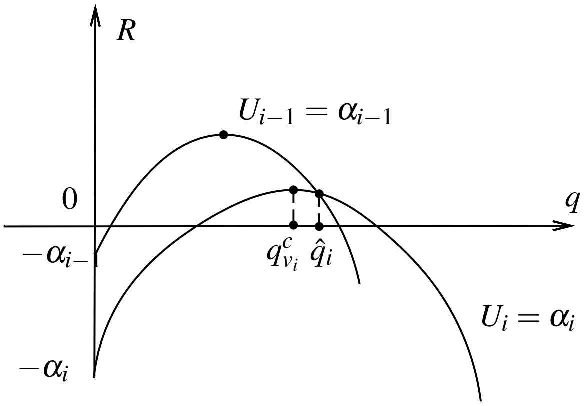

Figure 2: $\hat{q}_i$ is greater the optimal point $q^c_{v_i}$

4. Case of Two Types

It is worthwhile to consider optimal selling procedures for two buyer's types $v_1 < v_2$. Obviously $N(q^c_{v_1}, v_1) < N(q^c_{v_2}, v_2)$, i.e., the maximum of social surplus for the higher type is greater, but the maxima of the seller's profit for the two types of buyers are equal $N(q^c_{v_1}, v_1) - b_1$ and $N(q^c_{v_2}, v_2) - b_2$, hence cannot be compared a priori. Three cases of placement of the points depicting these maxima w.r.t. the respective indifference curves are possible.

(I) Each of the points $(q^c_{v_1}, N(q^c_{v_1}, v_1) - b_1)$ and $(q^c_{v_2}, N(q^c_{v_2}, v_2) - b_2)$ is not below the indifference type for the other type, i.e., $N(q^c_{v_1}, v_1) - b_1 \geq N(q^c_{v_1}, v_2) - b_2$ and $N(q^c_{v_2}, v_2) - b_2 \geq N(q^c_{v_2}, v_1) - b_1$ (Fig. 3).

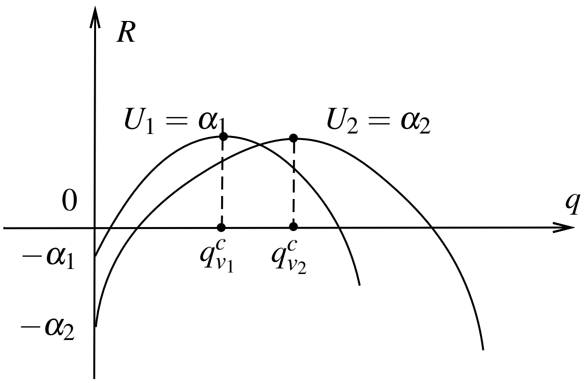

Figure 3: Optimal points are not below the indifference type

Then these points are optimal offer for the both types, and, moreover, the offers are effective, i.e., they provide the maximum of social surplus for each buyer's type.

(II) The point $(q^c_{v_1}, N(q^c_{v_1}, v_1) - b_1)$ of the maximum for the highest indifference curve for the lower type is below the highest indifference curve for the higher type, i.e., $N(q^c_{v_1}, v_1) - b_1 < N(q^c_{v_1}, v_2) - b_2$, which excludes the analogous inequality for the other point, and $N(q^c_{v_2}, v_2) - b_2 > N(q^c_{v_2}, v_1) - b_1$ is valid (Fig. 4). Draw an indifference curve for a buyer of the higher type through the point $(q^c_{v_1}, N(q^c_{v_1}, v_1) - b_1)$. It corresponds to the utility

$$\bar{b}_2 = b_2 + (N(q^c_{v_1}, v_2) - b_2) - (N(q^c_{v_1}, v_1) - b_1) = b_1 + N(q^c_{v_1}, v_2) - N(q^c_{v_1}, v_1).$$

Then the optimal selling procedure consists of the offer $(q^c_{v_2}, N(q^c_{v_2}, v_2) - b^*_2)$ for some $b^*_2 \in [b_2, \bar{b}_2)$, which is prepared for a higher type buyer, and the offer (q^*_1, R^*_1), for a lower type buyer, that is depicted by the intersection of the indifference curves of the level b_1 for the lower type and of the level b^*_2 for the higher type. This intersection is determined by the equality $N(q^*_1, v_1) - b_1 = N(q^*_1, v_2) - b^*_2$, where the both sides (i.e., the ordinate) must be non-negative. The expected seller's utility is equal to

$$\pi_1 (N(q^*_1, v_2) - b^*_2) + \pi_2 (N(q^c_{v_2}, v_2) - b^*_2) = \pi_1 N(q^*_1, v_2) + \pi_2 N(q^c_{v_2}, v_2) - b^*_2,$$

or, taking into account $b^*_2 = N(q^*_1, v_2) - N(q^*_1, v_1) + b_1$,

$$(\pi_1 - 1) N(q^*_1, v_2) + \pi_2 N(q^c_{v_2}, v_2) - N(q^*_1, v_1) - b_1 = \\ = \pi_2 N(q^c_{v_2}, v_2) - \pi_2 N(q^*_1, v_2) + N(q^*_1, v_1) - b_1.$$

The inequality $b_2 \le b^*_2 < \bar{b}_2$ becomes of the form

$$b_2 \le N(q^*_1, v_2) - N(q^*_1, v_1) + b_1 < b_1 + N(q^c_{v_1}, v_2) - N(q^c_{v_1}, v_1),$$

i.e.,

$$\begin{cases} N(q^*_1, v_2) - b_2 \ge N(q^*_1, v_1) - b_1, \\ N(q^*_1, v_2) - N(q^*_1, v_1) < N(q^c_{v_1}, v_2) - N(q^c_{v_1}, v_1), \end{cases}$$

and the second inequality is equivalent to $q^*_1 < q^c_{v_1}$, i.e., the lower type buyer's quantity must be subeffective.

Thus the problem

$$\begin{cases} \pi_1 N(q^*_1, v_2) + \pi_2 N(q^c_{v_2}, v_2) - b^*_2 \to \max, \\ N(q^*_1, v_1) - b_1 = N(q^*_1, v_2) - b^*_2 \ge 0, \\ b^*_2 \in \big[b_2, b_1 + N(q^c_{v_1}, v_2) - N(q^c_{v_1}, v_1)\big). \end{cases}$$

for an optimal selling procedure can be reduced to

$$\begin{cases} N(q_1^*, v_1) - \pi_2 N(q_1^*, v_2) \to \max, \\ N(q_1^*, v_2) - b_2 \geq N(q_1^*, v_1) - b_1 \geq 0, \\ q_1^* < q_{v_1}^c. \end{cases}$$

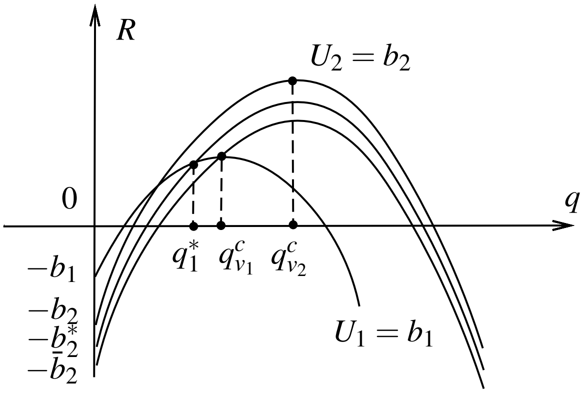

Figure 4: The point of the maximum for the highest indifference curve for the lower type is below the highest indifference curve for the higher type

Recall that the lower type's offer is not effective, but the higher type's offer can either be effective or be not effective.

(III) On the contrary, the point $(q_{v_2}^c, N(q_{v_2}^c, v_2) - b_2)$ is below the indifference curve for the lower type, i.e., $N(q_{v_2}^c, v_2) - b_2 < N(q_{v_2}^c, v_1) - b_1$, then $N(q_{v_1}^c, v_1) - b_1 > N(q_{v_1}^c, v_2) - b_1$ (Fig. 5). Quite analogously an optimal selling procedure is determined by the solution of a "mirror" problem

$$\begin{cases} N(q_2^*, v_2) - \pi_1 N(q_2^*, v_1) \to \max, \\ N(q_2^*, v_1) - b_1 \geq N(q_2^*, v_2) - b_2 \geq 0, \\ q_2^* > q_{v_2}^c. \end{cases}$$

Now the higher type's offer is not effective, and it is not known a priori whether the lower type's offer is effective.

5. Example Model Application

We show how to find an optimal selling procedure for a classical model example from [7], namely for vertically parallel demand. It is rather realistic and widely assumed than the marginal utility (i.e., the demand price) of

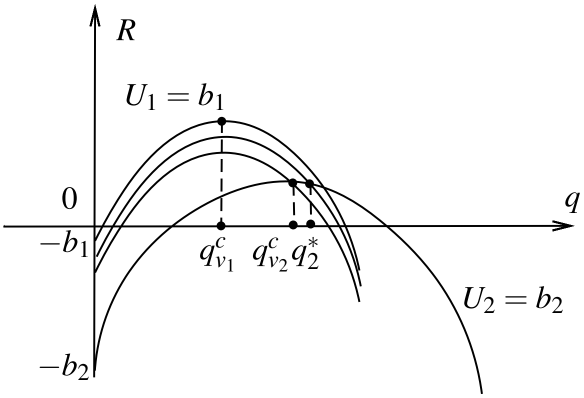

Figure 5: The point $(q^c_{v_2}, N(q^c_{v_2}, v_2) - b_2)$ is below the indifference curve for the lower type

the higher type is greater that the marginal utility for the lower price at a constant value for all product quantities.

Let the demand price be determined with the formula $p(q,v) = v - h(q)$, $h(0) \geq 0$, $h'(\cdot) > 0$, then $N(q,v) = q(v-c) - H(q)$, $H(q) = \int_0^q h(x)\,dx$, $H(0) = 0$, $H'(0) \geq 0$, $H''(\cdot) > 0$, hence the function $H(q)$ is increasing and convex. The equality $p(q^c_v) = c$ implies $q^c_v = h^{-1}(v-c)$, hence the type parameter v cannot be less than $c + h(0)$. For two types $v_1 < v_2$ the conditions $N(q^c_{v_1}, v_1) - b_1 \geq N(q^c_{v_1}, v_2) - b_2$ and $N(q^c_{v_2}, v_2) - b_2 \geq N(q^c_{v_2}, v_1) - b_1$ of effectiveness of the both offers (I) assume the form $q^c_{v_1}(v_1 - c) - b_1 \geq q^c_{v_1}(v_2 - c) - b_2$ and $q^c_{v_2}(v_2 - c) - b_2 \geq q^c_{v_2}(v_1 - c) - b_1$, i.e.,

$$q^c_{v_1} \leq \frac{b_2 - b_1}{v_2 - v_1} \leq q^c_{v_2} \iff v_1 \leq h\left(\frac{b_2 - b_1}{v_2 - v_1}\right) + c \leq v_2.$$

In particular, $b_2 \geq b_1$ is necessary, i.e., the higher type has not less requirements for minimal utility. In the right inequality $b_2 \geq b_1$ is implicit because the function h is not defined for negative arguments.

It $b_2 < b_1$ (which, of course, is illogical but left for the arguments completeness) or $h\left(\frac{b_2 - b_1}{v_2 - v_1}\right) + c < v_1$, i.e., the difference between b_1 is b_2 to small, then we obtain the case (II). Then the lower type receives the minimal acceptable utility, and the higher type's offer is equal to $q^c_{v_2}$. The respective problem for an optimal selling procedure is of the form

$$\begin{cases} q_1^*(v_1-c)-\pi_2 q_1^*(v_2-c)-\pi_1 H(q_1^*)\to \max, \\ q_1^*(v_2-c)-H(q_1^*)-b_2 \geq q_1^*(v_1-c)-H(q_1^*)-b_1 \geq 0, \\ h(q_1^*)<v_1-c, \end{cases}$$

$$\text{or } \begin{cases} q_1^*(v_1-\pi_1 c-\pi_2 v_2)-\pi_1 H(q_1^*)\to \max, \\ q_1^* \geq \dfrac{b_2-b_1}{v_2-v_1}, \\ q_1^* \geq \dfrac{b_1+H(q_1^*)}{v_1-c}, \\ h(q_1^*)<v_1-c. \end{cases}$$

The derivative of the objective function is equal to $v_1-\pi_1 c-\pi_2 v_2-\pi_1 h(q_1^*)$. Let q_1^0 be the greater of $\dfrac{b_2-b_1}{v_2-v_1}$ and the *least* of the solutions of the equation $q_1^*=\dfrac{b_1+H(q_1^*)}{v_1-c}$.

If $v_1-\pi_1 c-\pi_2 v_2 \leq \pi_1 h(q_1^0)$, then the objective function is decreasing for all arguments that satisfy the constraints, and the optimal offer for the lower type is determined by the quantity q_1^0. If, moreover, $q_1^0=\dfrac{b_2-b_1}{v_2-v_1}$, then the higher type's offer provides the minimal acceptable utility to the buyer.

Otherwise, if $v_1-\pi_1 c-\pi_2 v_2>\pi_1 h(q_1^0)$, then the optimal quantity $q_1^*>q_1^0$ to sell to a lower type buyer is the solution of the equation $\pi_1 h(q_1^*)=v_1-\pi_1 c-\pi_2 v_2$, his utility is the minimal acceptable one, and the higher type's utility is equal to his utility calculated for the lower type's offer.

What is left is to consider the case (III) when $h\left(\frac{b_2-b_1}{v_2-v_1}\right)+c>v_2$. Observe that this ensures $b_2>b_1$. We obtain analogously the optimization problem

$$\begin{cases} q_2^*(v_2-c)-\pi_1 q_2^*(v_1-c)-\pi_2 H(q_2^*)\to \max, \\ q_2^*(v_1-c)-H(q_2^*)-b_1 \geq q_2^*(v_2-c)-H(q_2^*)-b_2 \geq 0, \\ h(q_2^*)>v_2-c, \end{cases}$$

$$\text{or}\quad \begin{cases} q_2^*(v_2-\pi_2 c-\pi_2 v_1)-\pi_1 H(q_2^*)\to\max,\\ q_2^*\le \dfrac{b_2-b_1}{v_2-v_1},\\ q_2^*\ge \dfrac{b_2+H(q_2^*)}{v_2-c},\\ h(q_2^*)>v_2-c. \end{cases}$$

Let q_2^0 be the less of $\frac{b_2-b_1}{v_2-v_1}$ and the *greatest* of the solutions of the equation $q_2^*=\frac{b_2+H(q_2^*)}{v_2-c}$. If $v_2-\pi_2 c-\pi_2 v_2\ge \pi_2 h(q_2^0)$, then the objective function is increasing for the considered arguments, and q_2^0 is the optimal higher type's offer. If, moreover, $q_2^0=\frac{b_2-b_1}{v_2-v_1}$, then the lower type obtains the minimal acceptable utility, otherwise his utility is greater. For $v_2-\pi_2 c-\pi_2 v_2<\pi_2 h(q_2^0)$ we solve the equation $\pi_2 h(q_2^*)=v_2-\pi_2 c-\pi_2 v_1$ under constraint $h(q_2^*)>v_2-c$, and the following calculation of the optimal offers is analogous to the case (II). The utility of the lower type is minimal in all cases.

6. Conclusions

We observe that introduction of lower constraints on buyers' utilities significantly complicates the model even for two types and rather simple dependency of marginal utility on the buyer's type. Analysis of solutions for more buyers' types will not lead to so elegant characterization as in [7], although some conclusions can be drawn. In a subsequent paper we are going to describe the case of many buyer's types and investigate when ignoring some types is profitable. For price discrimination is not always advantageous [1], we will also establish conditions when the same offer can be associated with different types.

References

1. E.T. Anderson, J.D. Dana, *When Is Price Discrimination Profitable?* CSIO working paper, No. 0072 (2005), Center for the Study of Industrial Organization at Northwestern Univ., Evanston, Ill. https://hdl.handle.net/10419/38645
2. D. Bergemann, K. Schlag, *Robust Monopoly Pricing*, Cowles Foundation discussion paper (2007). doi:0.1.1.139.5961
3. G. Carroll, *When Are Local Incentive Constraints Sufficient?*, Econometrica, **80**:2 (2012), 661–686. doi:10.3982/ECTA9454
4. E. Damiano, H. Li, *Price Discrimination and Efficient Matching*, Economic Theory, **30**:2 (2007), 243–263. doi:10.1007/s00199-005-0058-2

5. S. Esteban, E. Miyagawa, M. Shum, *Nonlinear Pricing with Self-Control Preferences*, Journal of Economic Theory, **135**:1 (2007), 306–338. doi:10.1016/j.jet.2006.04.007
6. A.M. Manelli, D.R. Vincent, *Multidimensional Mechanism Design: Revenue Maximization and the Multiple-Good Monopoly Summary*, Fondazione Eni and Enrico Mattei (2004). https://ideas.repec.org/p/fem/femwpa/2004.153.html
7. E. Maskin, J. Riley, *Monopoly with Incomplete Information*, Rand Journal of Economics, **15**:2 (1984), 171–196. doi:10.2307/2555674
8. H. Shen, T. Basar, *Optimal Nonlinear Pricing for a Monopolistic Network Service Provider with Complete and Incomplete Information*, IEEE Journal on Selected Areas in Communications, **25**:6 (2007), 1216–1223. doi:10.1109/JSAC.2007.070815

Received by Editorial Board December 9, 2025
Accepted by Editorial Board February 10, 2026

МОДЕЛЬ НЕЛІНІЙНОГО ЦІНОУТВОРЕННЯ З ОБМЕЖЕНИМИ ЗНИЗУ КОРИСНОСТЯМИ ПОКУПЦЯ

І.В. Никифорчин*
Карпатський національний університет імені Василя Стефаника;
76018, вул. Шевченка 57, м. Івано-Франківськ, Україна
e-mail: iryna.nykyforchyn@cnu.edu.ua

Побудовано розширення класичної моделі теорії контрактів — моделі Маскіна і Райлі нелінійного ціноутворення. На корисність покупця накладено невід'ємне обмеження знизу і показано, що у цьому випадку не справджується твердження про те, що зі всіх умов сумісності винагороди суттєвими є тільки порівняння кожного типу покупців з попереднім нижчим типом, причому ці умови виконано як рівності. Доведено, що у нашій моделі необхідно і достатньо розглянути також порівняння з наступним вищим типом, а строгі нерівності у порівнянні деякого типу з усіма сусідами можливі тільки, якщо даний тип отримує мінімальну припустиму корисність. Запропоновано методи розв'язання даної задачі для двох типів покупців та вказано, коли пропозиції для обох чи одного з типів є ефективними (відповідають максимуму соціального надлишку).

Ключові слова: *Цінова дискримінація, продавець-монополіст, локальні обмеження, очікувана корисність.*