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DEGENERATE ORTHOGONALITY IN THE POLYNOMIAL SPACE INDUCED BY DISCRETE AND HERMITE-TYPE LOCAL FUNCTIONALS

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This paper studies degenerate bilinear forms on the space of polynomials generated by discrete functionals of Lagrange and Hermite type. It is shown that for a discrete form built from polynomial values at a finite set of nodes, its kernel coincides with the principal ideal generated by the polynomial vanishing at all nodes. Next, a Hermite-type generalization is established: if the form depends on derivative values up to prescribed orders at the nodes, and the corresponding local coefficient matrices are nondegenerate, then the kernel of the form coincides with the principal ideal generated by the product of the corresponding powers of linear factors. This makes it possible to interpret the quotient space of polynomials modulo the kernel as a space of finite jets, and the corresponding orthogonal decomposition as Lagrange or Hermite interpolation. In addition, a general theorem on local forms on jet spaces is formulated, covering Lagrange, Hermite, and mixed local schemes within a unified framework.

Key words: *degenerate orthogonality, polynomial ideals, Hermite interpolation, jet spaces, local bilinear forms, discrete orthogonal systems..*

1. Introduction

The classical theory of orthogonal polynomials usually relies on nondegenerate inner products of the form

$$\langle p,q\rangle = \int p(x)q(x)\,d\mu(x),$$

where μ is a positive measure on a subset of the real line. If the support of the measure is finite, then the corresponding form on the space of all

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polynomials becomes degenerate, and this naturally leads to the problem of describing its kernel and factoring the polynomial space by this kernel.

Although the discrete case is classical and closely related to Lagrange interpolation, it is useful to emphasize its algebraic meaning: the kernel of the form is a polynomial ideal in the ring $\mathbb{K}[x]$. This point of view becomes especially fruitful when passing to Hermite conditions, where the form depends not only on polynomial values at the nodes but also on the values of derivatives.

The aim of this paper is:

(1) to describe discrete degenerate forms on $\mathbb{K}[x]$ via their kernels;
(2) to show that these kernels are principal ideals;
(3) to construct natural orthogonal bases in the corresponding quotient spaces;
(4) to give a Hermite-type generalization and prove an exact description of the kernel;
(5) to interpret the quotient space in terms of jet spaces;
(6) to formulate a unifying principle for local bilinear forms on jet spaces.

Throughout the paper, $\mathbb{K}$ denotes either the field $\mathbb{R}$ or the field $\mathbb{C}$.

2. The Discrete Lagrange Setting

Let $x_1,\dots,x_m \in \mathbb{K}$ be pairwise distinct points, and let $w_1,\dots,w_m \in \mathbb{K}$ be nonzero weights. Consider on $\mathbb{K}[x]$ the bilinear form

$$B(p,q) = \sum_{i=1}^{m} w_i p(x_i) q(x_i).$$

Set $\omega(x) = \prod_{i=1}^{m}(x - x_i)$.

Definition 2.1. *The kernel of the form B is defined by*

$$\operatorname{Ker} B = \{p \in \mathbb{K}[x] \colon B(p,q) = 0 \text{ for all } q \in \mathbb{K}[x]\}.$$

Theorem 2.1. *Assume that $x_1,\dots,x_m$ are pairwise distinct and that $w_i \neq 0$ for all $i = 1,\dots,m$. Then*

$$\operatorname{Ker} B = (\omega(x)) = \{\omega(x)u(x) \colon u(x) \in \mathbb{K}[x]\}.$$

Proof. Let $p \in \operatorname{Ker} B$. For each $j \in \{1, \ldots, m\}$ consider the jth fundamental Lagrange polynomial

$$\ell_j(x) = \prod_{\substack{1 \le r \le m \\ r \ne j}} \frac{x - x_r}{x_j - x_r},$$

which satisfies $\ell_j(x_i) = \delta_{ij}$. Then

$$0 = B(p, \ell_j) = \sum_{i=1}^{m} w_i p(x_i) \ell_j(x_i) = w_j p(x_j).$$

Since $w_j \ne 0$, we obtain $p(x_j) = 0$ for all j. Hence p is divisible by $\omega(x)$, that is, $p \in (\omega(x))$.

Conversely, if $p(x) = \omega(x)u(x)$, then $p(x_i) = 0$ for all i, and therefore

$$B(p, q) = \sum_{i=1}^{m} w_i p(x_i) q(x_i) = 0$$

for every $q \in \mathbb{K}[x]$. Thus $p \in \operatorname{Ker} B$. The theorem is proved. □

Corollary 2.1. *The quotient space $\mathbb{K}[x]/(\omega(x))$ has dimension m, and each of its classes contains a unique representative of degree less than m.*

Proof. This follows immediately from polynomial division with remainder by $\omega(x)$. □

Proposition 2.1. *The fundamental Lagrange polynomials $\ell_1, \ldots, \ell_m$ form an orthogonal system with respect to B, and*

$$B(\ell_i, \ell_j) = w_i \delta_{ij}.$$

In particular, if $w_i > 0$, then the system

$$e_i(x) = \frac{\ell_i(x)}{\sqrt{w_i}}, \qquad i = 1, \ldots, m,$$

is an orthonormal basis of the quotient space $\mathbb{K}[x]/(\omega(x))$.

Proof. We have

$$B(\ell_i, \ell_j) = \sum_{k=1}^{m} w_k \ell_i(x_k) \ell_j(x_k) = \sum_{k=1}^{m} w_k \delta_{ik} \delta_{jk} = w_i \delta_{ij}.$$

The statement follows. □

Proposition 2.2. *For every polynomial $p \in \mathbb{K}[x]$, there exists a unique polynomial $q \in \mathbb{K}[x]$ such that*

$$p(x) = \sum_{i=1}^{m} p(x_i)\ell_i(x) + \omega(x)q(x). \tag{1}$$

Proof. Consider the polynomial

$$r(x) = p(x) - \sum_{i=1}^{m} p(x_i)\ell_i(x).$$

Then $r(x_j) = 0$ for all $j = 1, \dots, m$, hence r is divisible by $\omega(x)$. □

Remark 2.1. *Formula (1) shows that the orthogonal decomposition in the quotient space $\mathbb{K}[x]/(\omega(x))$ actually coincides with the Lagrange interpolation.*

3. Two-Point and Three-Point Examples

3.1. The two-point case $\{\pm a\}$

Let $a \in \mathbb{K}$, $a \neq 0$, and consider

$$B(p,q) = \alpha p(a)q(a) + \beta p(-a)q(-a),$$

where $\alpha, \beta \in \mathbb{K} \setminus \{0\}$. Then $\omega(x) = x^2 - a^2$, and by Theorem 2.1, $\operatorname{Ker} B = (x^2 - a^2)$.

Proposition 3.1. *If $\alpha + \beta \neq 0$, then the system $\phi_0(x) = 1$, $\phi_1(x) = x - \frac{a(\alpha-\beta)}{\alpha+\beta}$ is orthogonal with respect to B.*

Proof.

$$B(\phi_0, \phi_1) = \alpha\left(a - \frac{a(\alpha-\beta)}{\alpha+\beta}\right) + \beta\left(-a - \frac{a(\alpha-\beta)}{\alpha+\beta}\right) \tag{2}$$

$$= 0. \tag{3}$$

□

In the symmetric case $\alpha = \beta > 0$, we obtain the orthonormal basis

$$e_0(x) = \frac{1}{\sqrt{2\alpha}}, \qquad e_1(x) = \frac{x}{a\sqrt{2\alpha}}.$$

Every polynomial decomposes as

$$p(x) = A + Bx + (x^2 - a^2)q(x).$$

3.2. The three-point case $\{-a, 0, a\}$

Let $a \neq 0$ and consider

$$B(p,q) = p(-a)q(-a) + p(0)q(0) + p(a)q(a).$$

Then $\omega(x) = x(x^2 - a^2)$. The fundamental Lagrange polynomials are

$$\ell_{-a}(x) = \frac{x(x-a)}{2a^2}, \ \ell_0(x) = \frac{a^2 - x^2}{a^2}, \ \ell_a(x) = \frac{x(x+a)}{2a^2}.$$

The corresponding decomposition is

$$p(x) = p(-a)\ell_{-a}(x) + p(0)\ell_0(x) + p(a)\ell_a(x) + x(x^2 - a^2)q(x).$$

4. The Hermite-Type Generalization

We now take into account not only polynomial values at the nodes, but also values of derivatives.

Let $x_1, \dots, x_m \in \mathbb{K}$ be pairwise distinct points, and let $k_1, \dots, k_m \in \mathbb{N}$ be prescribed multiplicities. Set $N = k_1 + \dots + k_m$ and $\Omega(x) = \prod_{i=1}^{m}(x - x_i)^{k_i}$.

For each i, let $C^{(i)} = (c_{rs}^{(i)})_{r,s=0}^{k_i-1}$ be a square matrix of order k_i over $\mathbb{K}$.

Definition 4.1. *The Hermite-type bilinear form associated with the nodes x_i, multiplicities k_i, and matrices $C^{(i)}$ is defined by*

$$B_H(p,q) = \sum_{i=1}^{m} \sum_{r=0}^{k_i-1} \sum_{s=0}^{k_i-1} c_{rs}^{(i)} p^{(r)}(x_i) q^{(s)}(x_i). \tag{4}$$

Remark 4.1. *The simplest special case is the diagonal form*

$$B_H(p,q) = \sum_{i=1}^{m} \sum_{r=0}^{k_i-1} w_{i,r} p^{(r)}(x_i) q^{(r)}(x_i),$$

where all $w_{i,r} \neq 0$.

Theorem 4.1. *Assume that all matrices $C^{(i)}$ are nondegenerate. Then*

$$\operatorname{Ker} B_H = (\Omega(x)) = \{\Omega(x)u(x) \colon u(x) \in \mathbb{K}[x]\}.$$

Proof. First, we prove the inclusion $(\Omega(x)) \subseteq \operatorname{Ker} B_H$. If

$$p(x) = \Omega(x)u(x),$$

then at each point x_i the polynomial p has a zero of multiplicity at least k_i. Hence

$$p^{(r)}(x_i) = 0, \qquad r = 0, 1, \ldots, k_i - 1.$$

Therefore, all terms in (4) vanish, and $B_H(p,q) = 0$ for all $q \in \mathbb{K}[x]$. Thus $p \in \operatorname{Ker} B_H$.

Now let $p \in \operatorname{Ker} B_H$. For each i, consider the vector

$$v_i(p) = \big(p(x_i), p'(x_i), \ldots, p^{(k_i-1)}(x_i)\big)^T \in \mathbb{K}^{k_i}.$$

Then

$$B_H(p,q) = \sum_{i=1}^{m} v_i(p)^T C^{(i)} v_i(q).$$

Fix an index i. By Hermite interpolation, for every vector $u \in \mathbb{K}^{k_i}$ there exists a polynomial $q \in \mathbb{K}[x]$ such that $v_i(q) = u$, and for all $j \neq i$, $v_j(q) = 0$. Then the condition $B_H(p,q) = 0$ implies

$$v_i(p)^T C^{(i)} u = 0 \qquad \text{for all } u \in \mathbb{K}^{k_i}.$$

Hence, $v_i(p)^T C^{(i)} = 0$. Since $C^{(i)}$ is nondegenerate, it follows that $v_i(p) = 0$. Therefore,

$$p^{(r)}(x_i) = 0, \qquad r = 0, 1, \ldots, k_i - 1.$$

This holds for all i, so p is divisible by $\Omega(x)$. The theorem is proved. □

Corollary 4.1. *The quotient space $\mathbb{K}[x]/(\Omega(x))$ has dimension*

$$\dim \mathbb{K}[x]/(\Omega(x)) = N = \sum_{i=1}^{m} k_i,$$

and each of its classes contains a unique representative of degree less than N.

5. Hermite Interpolation as an Orthogonal Decomposition

The conditions

$$H^{(r)}(x_i) = y_{i,r}, \qquad i = 1, \ldots, m, \quad r = 0, \ldots, k_i - 1,$$

define a unique Hermite interpolation polynomial $H(x)$ of degree less than N.

Theorem 5.1. *For every polynomial $p \in \mathbb{K}[x]$, there exists a unique polynomial $q \in \mathbb{K}[x]$ such that*

$$p(x) = H_p(x) + \Omega(x)q(x), \tag{5}$$

where H_p is the unique polynomial of degree $< N$ satisfying

$$H_p^{(r)}(x_i) = p^{(r)}(x_i), \qquad i = 1, \dots, m, \quad r = 0, \dots, k_i - 1.$$

Proof. By the theorem on Hermite interpolation, there exists a unique polynomial H_p of degree $< N$ with the prescribed derivative values. Then the polynomial $p - H_p$ has a zero of multiplicity at least k_i at each node x_i, hence it is divisible by $\Omega(x)$. □

Remark 5.1. *Theorem 5.1 shows that the "visible" part of a polynomial in the quotient space $\mathbb{K}[x]/(\Omega(x))$ is precisely its Hermite interpolation polynomial.*

6. A Canonical Hermite Basis

For each $i = 1, \dots, m$ and $r = 0, \dots, k_i - 1$, there exists a unique polynomial $H_{i,r}(x)$ of degree $< N$ satisfying

$$H_{i,r}^{(s)}(x_j) = \delta_{ij}\delta_{rs}, \qquad j = 1, \dots, m, \quad s = 0, \dots, k_j - 1.$$

Proposition 6.1. *The system of polynomials*

$$\{H_{i,r}(x) \colon i = 1, \dots, m,\ r = 0, \dots, k_i - 1\}$$

forms a basis of the space of all polynomials of degree less than N. Moreover, for every $p \in \mathbb{K}[x]$, one has the expansion

$$p(x) = \sum_{i=1}^{m} \sum_{r=0}^{k_i - 1} p^{(r)}(x_i) H_{i,r}(x) + \Omega(x)q(x). \tag{6}$$

Proof. This is an immediate consequence of Theorem 5.1. □

Remark 6.1. *In the Lagrange case, when $k_i = 1$ for all i, the polynomials $H_{i,0}$ coincide with the fundamental Lagrange polynomials.*

7. Interpretation via Jet Spaces

For each i, consider the space of $(k_i - 1)$-jets at the point x_i, which in the one-dimensional polynomial case can be identified with the vector space

$$J_{x_i}^{k_i-1} = \mathbb{K}^{k_i}$$

with coordinates

$$\big(p(x_i), p'(x_i), \ldots, p^{(k_i-1)}(x_i)\big).$$

Then the mapping $T_H \colon \mathbb{K}[x] \to \bigoplus_{i=1}^{m} J_{x_i}^{k_i-1}$, defined by

$$T_H(p) = \big(v_1(p), \ldots, v_m(p)\big),$$

has kernel $\operatorname{Ker} T_H = (\Omega(x))$. Hence, $\mathbb{K}[x]/(\Omega(x)) \cong \bigoplus_{i=1}^{m} J_{x_i}^{k_i-1}$.

Remark 7.1. *Thus, the Hermite-type degenerate orthogonality on $\mathbb{K}[x]$ naturally transfers to a finite-dimensional jet space. This is one of the conceptual meanings of the construction.*

8. A General Theorem on Local Forms and Jet Spaces

In this section we formulate a statement that generalizes the previous results and conceptually reduces the Lagrange and Hermite cases to a single scheme.

For fixed pairwise distinct nodes $x_1, \ldots, x_m \in \mathbb{K}$ and multiplicities $k_1, \ldots, k_m \in \mathbb{N}$, let $\Omega(x) = \prod_{i=1}^{m}(x - x_i)^{k_i}$ and consider the space of local jets $J = \bigoplus_{i=1}^{m} J_{x_i}^{k_i-1} \cong \bigoplus_{i=1}^{m} \mathbb{K}^{k_i}$.

Theorem 8.1. *Let $\mathscr{B}$ be an arbitrary bilinear form on the space J. Then the formula*

$$B_{\mathscr{B}}(p,q) := \mathscr{B}\big(T_H(p), T_H(q)\big)$$

defines a bilinear form on $\mathbb{K}[x]$ such that

$$(\Omega(x)) \subseteq \operatorname{Ker} B_{\mathscr{B}}.$$

Moreover, the following statements are equivalent:

(1) the form $\mathscr{B}$ is nondegenerate on J;
(2) the induced form $B_{\mathscr{B}}$ on the quotient space $\mathbb{K}[x]/(\Omega(x))$ is nondegenerate;
(3)

$$\operatorname{Ker} B_{\mathscr{B}} = (\Omega(x)).$$

Proof. Since $T_H(p) = 0$ for every $p \in (\Omega(x))$, we have

$$B_{\mathscr{B}}(p,q) = \mathscr{B}(0, T_H(q)) = 0$$

for all $q \in \mathbb{K}[x]$. Hence, $(\Omega(x)) \subseteq \operatorname{Ker} B_{\mathscr{B}}$.

By construction, the operator T_H is surjective: for any prescribed set of local data in J, there exists a polynomial realizing these data by Hermite interpolation. Moreover, $\operatorname{Ker} T_H = (\Omega(x))$. Therefore, T_H induces an isomorphism

$$\widetilde{T}_H \colon \mathbb{K}[x]/(\Omega(x)) \to J.$$

If $\mathscr{B}$ is nondegenerate on J, then via the isomorphism $\widetilde{T}_H$ it transfers to a nondegenerate form on $\mathbb{K}[x]/(\Omega(x))$. This proves $1)\mathbb{R}ightarrow2)$.

The implication $2) \to 3)$ is immediate: if the induced form on the quotient is nondegenerate, then its kernel in $\mathbb{K}[x]$ must coincide with the factorization kernel, namely $(\Omega(x))$.

Finally, assume that $3)$ holds. Let $u \in J$ be such that

$$\mathscr{B}(u,v) = 0 \qquad \text{for all } v \in J.$$

Since T_H is surjective, there exists $p \in \mathbb{K}[x]$ such that $T_H(p) = u$. Then for every $q \in \mathbb{K}[x]$,

$$B_{\mathscr{B}}(p,q) = \mathscr{B}(T_H(p), T_H(q)) = \mathscr{B}(u, T_H(q)) = 0.$$

Thus $p \in \operatorname{Ker} B_{\mathscr{B}} = (\Omega(x))$, and hence $u = T_H(p) = 0$. Therefore, the form $\mathscr{B}$ is nondegenerate. The theorem is proved. □

Remark 8.1. *Theorem 8.1 shows that the whole construction is actually determined not by a specific formula on polynomials, but by the choice of a local bilinear form on the jet space. In particular, Theorem 4.1 is a special case in which $\mathscr{B}$ is given by a block-diagonal matrix composed of the matrices $C^{(i)}$.*

9. Additional Examples

Example 1 (One node of multiplicity 2)**.** *Let $a \in \mathbb{K}$ and consider*

$$B_H(p,q) = \alpha p(a)q(a) + \beta p'(a)q'(a), \qquad \alpha, \beta \neq 0.$$

Then

$$\Omega(x) = (x-a)^2, \qquad \operatorname{Ker} B_H = ((x-a)^2).$$

Every polynomial $p \in \mathbb{K}[x]$ has the unique decomposition

$$p(x) = p(a) + p'(a)(x-a) + (x-a)^2 q(x).$$

Example 2 (One node of multiplicity 3)**.** *Fix a point $a \in \mathbb{K}$ and consider the form*

$$B_H(p,q) = \alpha_0 p(a)q(a) + \alpha_1 p'(a)q'(a) + \alpha_2 p''(a)q''(a), \qquad \alpha_0, \alpha_1, \alpha_2 \neq 0.$$

Then

$$\Omega(x) = (x-a)^3, \qquad \operatorname{Ker} B_H = ((x-a)^3).$$

Every polynomial $p \in \mathbb{K}[x]$ has the unique decomposition

$$p(x) = p(a) + p'(a)(x-a) + \frac{p''(a)}{2}(x-a)^2 + (x-a)^3 q(x).$$

Example 3 (Two nodes of multiplicities 2 and 1)**.** *Let $a, b \in \mathbb{K}$ be two distinct nodes and consider the form*

$$B_H(p,q) = \alpha_0 p(a)q(a) + \alpha_1 p'(a)q'(a) + \beta p(b)q(b), \qquad \alpha_0, \alpha_1, \beta \neq 0.$$

Then

$$\Omega(x) = (x-a)^2(x-b), \qquad \operatorname{Ker} B_H = ((x-a)^2(x-b)).$$

For every polynomial p one has the decomposition

$$p(x) = p(a)H_{a,0}(x) + p'(a)H_{a,1}(x) + p(b)H_{b,0}(x) + (x-a)^2(x-b)q(x),$$

where the basis polynomials are determined by the conditions

$$H_{a,0}(a) = 1, \qquad H'_{a,0}(a) = 0, \qquad H_{a,0}(b) = 0, \tag{7}$$

$$H_{a,1}(a) = 0, \qquad H'_{a,1}(a) = 1, \qquad H_{a,1}(b) = 0, \tag{8}$$

$$H_{b,0}(a) = 0, \qquad H'_{b,0}(a) = 0, \qquad H_{b,0}(b) = 1. \tag{9}$$

Example 4 (A nondiagonal local form)**.** *Let $m = 1$, $x_1 = a$, $k_1 = 2$, and let the local form matrix be*

$$C = \begin{pmatrix} \alpha & \gamma \\ \gamma & \beta \end{pmatrix}, \qquad \alpha\beta - \gamma^2 \neq 0.$$

Then

$$B_H(p,q) = \alpha p(a)q(a) + \gamma p(a)q'(a) + \gamma p'(a)q(a) + \beta p'(a)q'(a).$$

Since the matrix C is nondegenerate, we have $\operatorname{Ker} B_H = ((x-a)^2)$. This example shows that the structure of the kernel is determined not by diagonality of the form, but by the completeness of the local jet information at the node.

10. Conclusions

A unified scheme for Lagrange and Hermite-type degenerate forms on the polynomial space has been constructed. It has been shown that:

(1) in the discrete Lagrange case, the kernel of the form coincides with the ideal $(\omega(x))$, where ω vanishes at all nodes;
(2) in the Hermite case, under the nondegeneracy of the local matrices, the kernel of the form coincides with the ideal $(\Omega(x))$, where

$$\Omega(x) = \prod_{i=1}^{m} (x - x_i)^{k_i};$$

(3) the corresponding quotient spaces admit a natural interpolation interpretation;
(4) the Hermite setting is naturally described in the language of finite jets;
(5) the general theorem on local forms on jet spaces covers Lagrange, Hermite, and mixed local schemes within a single framework.

These results may serve as a basis for further studies of degenerate orthogonal structures related to local functionals, generalized functions, and polynomial modules.

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ВИРОДЖЕНА ОРТОГОНАЛЬНІСТЬ У ПРОСТОРІ ПОЛІНОМІВ, ІНДУКОВАНОМУ ДИСКРЕТНИМИ ТА ЕРМІТОВОГО ТИПУ ЛОКАЛЬНИМИ ФУНКЦІОНАЛАМИ

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У цій статті досліджуються вироджені білінійні форми на просторі поліномів, породжених дискретними функціоналами типу Лагранжа та Ерміта. Показано, що для дискретної форми, побудованої на основі значень полінома в скінченній множині вузлів, її ядро збігається з головним ідеалом, породженим поліномом, що занулюється у всіх вузлах. Далі встановлено узагальнення типу Ерміта: якщо форма залежить від значень похідних заданих порядків у вузлах, а відповідні локальні матриці коефіцієнтів є невиродженими, то ядро форми збігається з головним ідеалом, породженим добутком відповідних степенів лінійних множників. Це дає змогу інтерпретувати фактор-простір поліномів за модулем ядра як простір скінченних струменів, а відповідний ортогональний розклад — як інтерполяцію Лагранжа або Ерміта. Крім того, сформульовано загальну теорему про локальні форми на просторах струменів, яка охоплює схеми Лагранжа, Ерміта та змішані локальні схеми в єдину систему.

Ключові слова: *вироджена ортогональність, поліномні ідеали, ермітова інтерполяція, простори струменів, локальні білінійні форми, дискретна ортогональна система.*