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ON SOME COUNTABLY GENERATED ALGEBRA OF ENTIRE FUNCTIONS OF BOUNDED TYPE ON THE SPACE OF ALL BOUNDED SEQUENCES OF COMPLEX NUMBERS AND ITS SPECTRUM

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Let ℓ_∞ be the complex Banach space of all bounded sequences of complex numbers. In this paper, we study the Fréchet subalgebra $H_{b\mathscr{P}}(\ell_\infty)$ of the Fréchet algebra of all entire functions of bounded type $H_b(\ell_\infty)$, generated by some fixed countable set $\mathscr{P}$ of continuous algebraically independent complex-valued homogeneous polynomials on the space ℓ_∞ such that the set $\mathscr{P}$ contains an infinite number of elements that sharing the same degree of homogeneity. We investigate the form of elements of this subalgebra. Furthermore, we show that every linear multiplicative functional, acting from $H_{b\mathscr{P}}(\ell_\infty)$ to $\mathbb{C}$, can be uniquely identified with the sequence of its values on the elements of $\mathscr{P}$. This sequence has at most growth of some geometric progression. We use these results to describe the spectrum of the algebra $H_{b\mathscr{P}}(\ell_\infty)$.

Key words: *n-homogeneous polynomial, symmetric polynomial, analytic function, spectrum of algebra.*

1. Introduction

The problem of describing the spectrum of the Fréchet algebra $H_b(X)$ of entire functions of bounded type on a complex Banach space X is among the central open questions in nonlinear functional analysis and remains unresolved in the general setting (see [1], [27]). However, for certain subalgebras of $H_b(X)$ the spectral description is simplified. Important examples include subalgebras consisting of symmetric elements of $H_b(X)$ when the space X has a symmetric structure. For many such algebras explicit spectral descriptions have been obtained (see [3], [4], [5], [24]), largely due to the fact that these algebras are finitely or countably generated, meaning that their dense subalgebras of symmetric continuous polynomials have finite or countable algebraic bases. In particular, the algebras of continuous polynomials on spaces such as ℓ_p, where $p \in [1, +\infty)$, and $L_\infty[0,1]$ have algebraic bases consisting of homogeneous polynomials with exactly one polynomial for each degree of homogeneity (see [4], [6], [14]). These types of algebras have been studied in a broader context in [7], [8], [15], [18]. For algebras of symmetric polynomials on Cartesian powers of certain Banach spaces (see [19], [20], [21], [22], [23], [25], [26]) or for algebras of so-called block-symmetric polynomials (see [2], [9], [10], [11], [12], [28]) the algebraic bases may include multiple polynomials of the same degree of homogeneity. In all of the above-mentioned cases, there is a finite number of polynomials of the same degree of homogeneity in the algebraic bases.

In contrast to these algebras, the present work considers the algebra generated by the family of polynomials that contains an infinite number of n-homogeneous polynomials for every $n \in \mathbb{N}$. Specifically, in this paper we investigate the Fréchet subalgebra $H_{b\mathscr{P}}(\ell_\infty)$ of the Fréchet algebra $H_b(\ell_\infty)$, generated by some fixed countable set

$$\mathscr{P} = \big(P_{11}, P_{21}, P_{22}, P_{31}, P_{32}, P_{33}, \ldots, P_{n1}, P_{n2}, \ldots, P_{nn}, \ldots\big)$$

of continuous algebraically independent complex-valued k-homogeneous polynomials P_{jk} on a complex Banach space ℓ_∞, where $j \in \mathbb{N}$ and $k \in \{1, \ldots, j\}$, that is, for every degree of homogeneity the set $\mathscr{P}$ contains an infinite number of polynomials. We investigate the form in which the elements of this subalgebra can be represented. Furthermore, we show that every linear multiplicative functional φ, acting from $H_{b\mathscr{P}}(\ell_\infty)$ to $\mathbb{C}$, is completely determined by its values on the elements of $\mathscr{P}$, that is, φ can be uniquely identified with the sequence

$$\xi = \big(\varphi(P_{11}), \varphi(P_{21}), \varphi(P_{22}), \ldots, \varphi(P_{n1}), \varphi(P_{n2}), \ldots, \varphi(P_{nn}), \ldots\big).$$

We prove that the sequence ξ grows no faster than some geometric progression. We also describe the spectrum of the algebra $H_{b\mathscr{P}}(\ell_\infty)$.

2. Preliminaries

Let us denote by $\mathbb{N}$ the set of all positive integers, by $\mathbb{Z}^+$ the set of all nonnegative integers and by $\mathbb{Q}^+$ the set of all nonnegative rational numbers.

Polynomials on Banach spaces. Let X be a complex Banach space. Let a mapping $P : X \to \mathbb{C}$ be such that there exist $n \in \mathbb{N}$ and some n-linear form

$A_P : X^n \to \mathbb{C}$ such that $P(x) = A_P(\underbrace{x,...,x}_{n})$ for every $x \in X$. Then the mapping P is called an *n-homogeneous polynomial*.

A mapping $P : X \to \mathbb{C}$ is said to be a *polynomial of degree at most n*, where $n \in \mathbb{Z}^+$, if it can be represented as $P = P_0 + P_1 + ... + P_n$, where P_j is a j-homogeneous polynomial acting from X to $\mathbb{C}$ for every $j = \overline{1,n}$ and P_0 is a constant mapping acting from X to $\mathbb{C}$.

For each polynomial $P : X \to \mathbb{C}$ we put

$$\|P\| = \sup\{|P(x)| : x \in X, \|x\| \le 1\}. \tag{1}$$

It is known that a polynomial $P : X \to \mathbb{C}$ is continuous if and only if $\|P\| < \infty$.

Polynomials $P_1, P_2, \ldots$ that act from X to $\mathbb{C}$ are called *algebraically independent polynomials* when the following condition is satisfied: for every $n \in \mathbb{N}$ and for every polynomial $q : \mathbb{C}^n \to \mathbb{C}$ if

$$q\big(P_1(x), P_2(x), \ldots, P_n(x)\big) = 0$$

for every $x \in X$, then $q \equiv 0$.

A polynomial $P : X \to \mathbb{C}$ is called an *algebraic combination* of elements of the set $\mathbb{P} = \{P_1, P_2, \ldots\}$ if there exist $n \in \mathbb{N}$ and a polynomial $q : \mathbb{C}^n \to \mathbb{C}$ such that $P(x) = q\big(P_1(x), \ldots, P_n(x)\big)$ for every $x \in X$.

Let A be some subalgebra of the algebra of all continuous polynomials on X. Finite or countable subset $G \subset A$ is called an *algebraic basis* of the algebra A if the elements from the set G are algebraically independent and every element of the algebra A can be represented as an algebraic combination of the elements of G. As a result of algebraic independence of the elements of the set G this representation is unique.

Spectrum of an algebra. Let $A(T)$ be a topological algebra of some functions on a nonempty set T over a field $\mathbb{C}$. A nontrivial continuous linear multiplicative functional $\varphi : A \to \mathbb{C}$ is called a character of the algebra A. The set of all characters of the algebra A is called the *spectrum* of the algebra A. For $x \in T$ let $\delta_x : A(T) \to \mathbb{C}$ be defined by $\delta_x(f) = f(x)$, where $f \in A(T)$. The mapping δ_x is called a *point-evaluation functional*. Note that δ_x is linear and multiplicative.

The algebra $H_b(X)$. Let X be a complex Banach space. Let $H_b(X)$ be the Fréchet algebra of all entire functions $f : X \to \mathbb{C}$ which are bounded on bounded sets endowed with the topology of uniform convergence on bounded sets. For $f \in H_b(X)$ and $r > 0$ we put

$$\|f\|_r = \sup_{\|x\| \le r} \big|f(x)\big|.$$

The topology of $H_b(X)$ can be generated by an arbitrary set of norms $\big\{\|\cdot\|_r : r \in \Gamma\big\}$, where Γ is any unbounded subset of $(0, +\infty)$.

3. Main results

3.1. On the algebra $H_{b\mathscr{P}}(\ell_\infty)$

Let ℓ_∞ be a complex Banach space of all bounded sequences of complex numbers. Let

$$A = \big\{(j,k) \in \mathbb{N} \times \mathbb{N} : j \ge k\big\}. \tag{2}$$

Let us define the mapping $\tau : A \to \mathbb{N}$ by the formula

$$\tau\left((j,k)\right) = \frac{j(j-1)}{2} + k. \tag{3}$$

It can be checked that the mapping τ, defined by (3), is a bijection. Consequently, for every $x = (x_1, x_2, \ldots) \in \ell_\infty$ one has $\sup_{i\in\mathbb{N}} |x_i| = \sup_{\substack{j\in\mathbb{N}\\ k=\overline{1,j}}} |x_{\tau((j,k))}|$, that is,

$$\|x\| = \sup_{\substack{j\in\mathbb{N}\\ k=\overline{1,j}}} |x_{\tau((j,k))}|. \tag{4}$$

Let $P_{jk} : \ell_\infty \to \mathbb{C}$ be defined by

$$P_{jk}(x) = x^k_{\tau((j,k))} \tag{5}$$

for $x = (x_1, x_2, \ldots) \in \ell_\infty$, where $j \in \mathbb{N}$, $k \in \{1, 2, \ldots, j\}$ and $\tau\left((j,k)\right)$ is defined by (3). For example,

$$P_{11}(x) = x_1,$$
$$P_{21}(x) = x_2, P_{22}(x) = x_3^2,$$
$$P_{31}(x) = x_4, P_{32}(x) = x_5^2, P_{33}(x) = x_6^3$$

for $x = (x_1, x_2, \ldots) \in \ell_\infty$. Clearly, P_{jk} is a k-homogeneous polynomial. For $j \in \mathbb{N}$ and $k \in \{1, 2, \ldots, j\}$, by (1) and (5),

$$\|P_{jk}\| = \sup_{\|x\|\le 1} |P_{jk}(x)| = \sup_{\|x\|\le 1} |x^k_{\tau((j,k))}| = 1, \tag{6}$$

and so P_{jk} is continuous. Let

$$\mathscr{P} = \big(P_{11}, P_{21}, P_{22}, P_{31}, P_{32}, P_{33}, \ldots, P_{n1}, P_{n2}, \ldots, P_{nn}, \ldots\big). \tag{7}$$

Since the mapping τ is injective, it follows that the elements of the sequence (7) are algebraically independent.

Let us denote by $P_{\mathscr{P}}(\ell_\infty)$ the algebra of all polynomials which are algebraic combinations of elements of the set $\mathscr{P}$. Note that the elements of the algebra $P_{\mathscr{P}}(\ell_\infty)$ are symmetric polynomials in the following sense. For $n \in \mathbb{N}$ let us define $(j_n, k_n) \in A$ as $(j_n, k_n) = \tau^{-1}(n)$, where A is defined by (2) and τ is defined by (3). Let $g_{(s_1, s_2, \ldots)} : \ell_\infty \to \ell_\infty$, where $s_n \in \{1, \ldots, k_n\}$ for $n \in \mathbb{N}$, be defined by

$$g_{(s_1,s_2,\ldots)}\big((x_1, x_2, \ldots, x_n, \ldots)\big) \\ = \left(x_1 \exp\left(\frac{2\pi i s_1}{k_1}\right), x_2 \exp\left(\frac{2\pi i s_2}{k_2}\right), \ldots, x_n \exp\left(\frac{2\pi i s_n}{k_n}\right), \ldots\right),$$

where $(x_1, x_2, \ldots) \in \ell_\infty$. Let

$$G = \big\{g_{(s_1,s_2,\ldots)} : s_1 \in \{1, \ldots, k_1\}, s_2 \in \{1, \ldots, k_2\}, \ldots\big\}.$$

Every $P \in P_{\mathscr{P}}(\ell_\infty)$ is G-symmetric, i.e.

$$P\big(g(x)\big) = P(x)$$

for every $g \in G$ and $x \in \ell_\infty$.

Let us denote by $H_{b\mathscr{P}}(\ell_\infty)$ the Fréchet subalgebra of the Fréchet algebra $H_b(\ell_\infty)$ that consists of all entire functions of bounded type on ℓ_∞ such that the terms of the Taylor series of these functions are elements of the algebra $P_{\mathscr{P}}(\ell_\infty)$.

Let the set A is defined by (2). Let us denote by Ω any finite subset of A. Let

$$l : \Omega \to \mathbb{N} \tag{8}$$

be an arbitrary mapping. Let

$$\varkappa(\Omega, l) = \sum_{(j,k)\in\Omega} k \cdot l\big((j,k)\big). \tag{9}$$

Then the following theorem is true.

Theorem 3.1. *Every function $f \in H_{b\mathscr{P}}(\ell_\infty)$ can be uniquely represented in the form*

$$f(x) = \alpha_0 + \sum_{n=1}^{\infty} \sum_{\substack{\Omega \subset A \\ |\Omega| < \infty}} \sum_{\substack{l:\Omega \to \mathbb{N} \\ \varkappa(\Omega,l)=n}} \alpha_{(\Omega,l)} \prod_{(j,k)\in\Omega} P_{jk}^{l((j,k))}(x), \tag{10}$$

where $x \in \ell_\infty$, $\alpha_0, \alpha_{(\Omega,l)} \in \mathbb{C}$, A is defined by (2)*, the mapping l is defined by* (8) *and $\varkappa(\Omega,l)$ is defined by* (9).

Proof. Let $f \in H_{b\mathscr{P}}(\ell_\infty)$. Let $\alpha_0 = f(0)$. Let $n \in \mathbb{N}$ and let the polynomial f_n be the n^{th} term of the Taylor series of the function f. Then, by the definition of the algebra $H_{b\mathscr{P}}(\ell_\infty)$, $f_n \in P_{\mathscr{P}}(\ell_\infty)$, that is, f_n can be represented as an algebraic combination of elements of the sequence $\mathscr{P}$, defined by (7), that is, in the form

$$f_n(x) = \sum_{\substack{\Omega \subset A \\ |\Omega| < \infty}} \sum_{\substack{l:\Omega \to \mathbb{N} \\ \varkappa(\Omega,l)=n}} \alpha_{(\Omega,l)} \prod_{(j,k)\in\Omega} P_{jk}^{l\left((j,k)\right)}(x),$$

where $x \in \ell_\infty$, $\alpha_{(\Omega,l)} \in \mathbb{C}$, the set A is defined by (2), the mapping l is defined by (8) and $\varkappa(\Omega,l)$ is defined by (9). This representation is unique, since the elements of the sequence $\mathscr{P}$ are algebraically independent. This completes the proof of the theorem. □

Let us denote by $M_{b\mathscr{P}}$ the spectrum of the algebra $H_{b\mathscr{P}}(\ell_\infty)$. According to Theorem 3.1, every function $f \in H_{b\mathscr{P}}(\ell_\infty)$ can be uniquely represented in the form (10). Therefore, for every character $\varphi \in M_{b\mathscr{P}}$,

$$\varphi(f) = \alpha_0 + \sum_{n=1}^{\infty} \sum_{\substack{\Omega \subset A \\ |\Omega| < \infty}} \sum_{\substack{l:\Omega \to \mathbb{N} \\ \varkappa(\Omega,l)=n}} \alpha_{(\Omega,l)} \prod_{(j,k)\in\Omega} \left(\varphi(P_{jk})\right)^{l((j,k))},$$

since φ is continuous, linear and multiplicative, and $\varphi(1) = 1$. Thus, we see that every character $\varphi \in M_{b\mathscr{P}}$ is completely defined by its values on the polynomials P_{jk}, where $j \in \mathbb{N}$ and $k \in \{1, \ldots, j\}$. Hence, every character $\varphi \in M_{b\mathscr{P}}$ can be uniquely identified with the sequence

$$\big(\varphi(P_{11}), \varphi(P_{21}), \varphi(P_{22}), \ldots, \varphi(P_{n1}), \varphi(P_{n2}), \ldots, \varphi(P_{nn}), \ldots\big). \tag{11}$$

For every linear continuous functional $\varphi \in \left(H_{b\mathscr{P}}(\ell_\infty)\right)^*$ there exists $r \in \mathbb{Q}^+$ such that the functional φ is continuous with respect to the norm

$\|\cdot\|_r$, where $\left(H_{b\mathscr{P}}(\ell_\infty)\right)^*$ is a space of all continuous linear functionals on the algebra $H_{b\mathscr{P}}(\ell_\infty)$.

Let us define the radius function on the space $\left(H_{b\mathscr{P}}(\ell_\infty)\right)^*$ similar to [1, p. 53]. We define the *radius function* R on $\left(H_{b\mathscr{P}}(\ell_\infty)\right)^*$ by declaring $R(\varphi)$ to be the infimum of all $r \in \mathbb{Q}^+$ such that φ is continuous with respect to the norm $\|\cdot\|_r$. Thus, $0 \le R(\varphi) < \infty$.

Theorem 3.2. *For every character $\varphi \in M_{b\mathscr{P}}$ there exists $r \in \mathbb{Q}^+$ such that the estimate*

$$|\varphi(Q)| \le r^k \|Q\|_1$$

holds for every k-homogeneous polynomial Q belonging to the algebra $H_{b\mathscr{P}}(\ell_\infty)$.

Proof. Let $\varphi \in M_{b\mathscr{P}}$ and $R(\varphi)$ be the radius function of the character φ. Since φ is continuous, it follows that $0 \le R(\varphi) < \infty$. Let $r > R(\varphi)$. Then, since the norm of each nonzero continuous complex-valued homomorphism is equal to 1, the estimate $|\varphi(Q)| \le \|Q\|_r$ holds for every k-homogeneous polynomial Q that belongs to the algebra $H_{b\mathscr{P}}(X)$. Thus,

$$|\varphi(Q)| \le \sup_{\|x\| \le r} |Q(x)|. \tag{12}$$

Substituting $x = ry$ into the right hand side of (12) and taking into account that Q is a k-homogeneous polynomial we obtain

$$\sup_{\|x\| \le r} |Q(x)| = \sup_{\|ry\| \le r} |Q(ry)| = \sup_{r\|y\| \le r} r^k |Q(y)| = r^k \sup_{\|y\| \le 1} |Q(y)| = r^k \|Q\|_1. \tag{13}$$

By (12) and (13),

$$|\varphi(Q)| \le r^k \|Q\|_1.$$

This completes the proof of the theorem. □

Corollary 3.1. *For every character $\varphi \in M_{b\mathscr{P}}$ there exists $r \in \mathbb{Q}^+$ such that the estimate $|\varphi(P_{jk})| \le r^k$ holds for every polynomial P_{jk}, defined by* (5)*, where $j \in \mathbb{N}$ and $k \in \{1, \ldots, j\}$, that is, the sequence* (11) *grows no faster than some geometric progression.*

Proof. Theorem 3.2 and the equality (6) imply the statement of the corollary. □

Theorem 3.3. *The spectrum $M_{b\mathscr{P}}$ of the algebra $H_{b\mathscr{P}}(\ell_\infty)$ coincides with the set of all point-evaluation functionals at points of the space ℓ_∞.*

Proof. Let $\varphi \in M_{b\mathscr{P}}$. Let δ_x be a point-evaluation functional at some point $x \in \ell_\infty$. Let us show that $\varphi = \delta_x$ for some $x \in \ell_\infty$. As stated above, every character $\varphi \in M_{b\mathscr{P}}$ is uniquely identified by the sequence (11). Let

$$x = \left(\varphi(P_{11}), \varphi(P_{21}), \sqrt{\varphi(P_{22})}, \ldots, \varphi(P_{n1}), \sqrt{\varphi(P_{n2})}, \ldots, \sqrt[n]{\varphi(P_{nn})}, \ldots\right), \tag{14}$$

where $\sqrt[n]{z} = \sqrt[n]{|z|}\exp\left(\frac{i\arg z}{n}\right)$ for $z \in \mathbb{C}$.

Corollary (3.1) implies there exists $r \in \mathbb{Q}^+$ such that $|\sqrt[k]{\varphi(P_{jk})}| \le r$ for every polynomial P_{jk}, defined by (5), where $j \in \mathbb{N}$ and $k \in \{1, \ldots, j\}$. Therefore, $x \in \ell_\infty$. Furthermore, for all $j \in \mathbb{N}$ and $k \in \{1, \ldots, j\}$, by (5), the following equality holds:

$$\delta_x(P_{jk}) = P_{jk}(x) = x^k_{\tau((j,k))} = \left(\sqrt[k]{\varphi(P_{jk})}\right)^k = \varphi(P_{jk}). \tag{15}$$

Then from (15) it follows that $\varphi = \delta_x$. □

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ПРО ДЕЯКУ ЗЛІЧЕННО ПОРОДЖЕНУ АЛГЕБРУ ЦІЛИХ ФУНКЦІЙ ОБМЕЖЕНОГО ТИПУ НА ПРОСТОРІ УСІХ ОБМЕЖЕНИХ ПОСЛІДОВНОСТЕЙ КОМПЛЕКСНИХ ЧИСЕЛ ТА ЇЇ СПЕКТР

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Нехай ℓ_∞ є комплексним банаховим простором усіх обмежених послідовностей комплексних чисел. У даній роботі досліджено підалгебру Фреше $H_{b\mathscr{P}}(\ell_\infty)$ алгебри Фреше всіх цілих функцій обмеженого типу $H_b(\ell_\infty)$, породжену деякою фіксованою зліченною множиною $\mathscr{P}$ неперервних алгебраїчно незалежних комплекснозначних однорідних поліномів на комплексному банаховому просторі ℓ_∞, такою, що множина $\mathscr{P}$ містить нескінченну кількість елементів, які мають однаковий степінь однорідності. Встановлено вигляд елементів цієї підалгебри. Крім того, показано, що кожний лінійний мультиплікативний функціонал, який діє з алгебри $H_{b\mathscr{P}}(\ell_\infty)$ у простір $\mathbb{C}$, можна єдиним чином ідентифікувати з послідовністю його значень на елементах множини $\mathscr{P}$ і доведено, що ця послідовність зростає не швидше, ніж деяка геометрична прогресія. Ці результати використано для опису спектру алгебри $H_{b\mathscr{P}}(\ell_\infty)$.

Ключові слова: *n-однорідний поліном, симетричний поліном, аналітична функція, спектр алгебри.*