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ON ONE NONLINEAR MATHEMATICAL MODEL OF BLOOD CIRCULATION WITH THE VESSEL WALLS REACTION WITHIN THE HEREDITARY THEORY

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The research demonstrates sufficient conditions of the existence and uniqueness for the solution in the oscillation mathematical model of the blood flow under nonlinear dissipative forces action within the theory of hereditary tube with biofactor. The obtained qualitative results advocate the application of Galerkin method to the above-mentioned problem. These results facilitated the application of different (explicit and implicit) numerical methods in further studies of the dynamical characteristics of solutions in the considered oscillation mathematical models. Numerical integration of the movement equations by Runge-Kutta 4th order method and Geer 2nd order method in a model case within this research enabled the estimation of the influence of different physical and mechanical factors on the amplitude and frequency of the oscillation process. The use of hybrid methods for the oscillation modeling in the nonlinear isotropic elastic environment on the example of a vessel enabled the formulation of the equation of an object's mechanical state based on energy approaches and the theory of mechanical fields in the continuous environments.

Keywords*: mathematical model, nonlinear vibrations, Galerkin method, biofactor, blood circulation, vessel.*

1 Introduction

Modern social development trends such as the problems of human survival and healthy lifestyle preservation are closely interconnected with the general human problems. Under such circumstances, top priority tasks are to improve the quality of human life, to devise a formula for active longevity, and to raise the individual living standards. Physical and spiritual human self-awareness should be also considered. That is why the problem of an adequate mathematical modeling of the processes in living organisms is the problem of current interest for the modern healthcare and science in general. A lot of actual material that was collected by practitioners and enriched by modern hardware investigations attracts close attention of experts in numerical modeling. Processes providing human vital activity are so complicated and connected that the liaison between mathematicians, biologists and doctors is insistent demand for successful results. Such collaboration allows providing insights into regularities of human organism functioning as a unity, and as a result, assists an increase in human active lifestyle. At the same time computer modeling is progressing, and the requirements for mathematical models describing these processes are increasing. Almost all main organs and human organism systems become the object of research in modern physical modeling. Other areas of interest for researchers are the processes at the cellular and genetic levels. Many mechanisms of disease occurrence and course are being studied: wounds healing; oncological processes; immunological issues; drug delivery; creation and functioning of the artificial organs. Mathematical models of many organs and body parts – skin, bones, muscles, circulatory system etc, – are based on mechanical models which are well-known from the mechanics of deformed solids. Hydrodynamic staging based on the Navier-Stokes equations appear in hemodynamic problems, functioning of respiratory and digestive organs, etc. The approaches based on the diffusion reactions and thermal conductivity equations comprise a significant part of mathematical models of thrombosis, stomach and skin functioning, and disease treatment by thermotherapy and chemotherapy. Differential equations systems form the basis of the blood circulation model, nerve impulses transmission, cellular interactions and gene networks functioning. Hybrid models, which present all aspects of medical and biological processes in their correlation, become more and more popular.

The spectrum of the considered problems is so wide that the whole investigation review is almost impossible. Brief but sufficiently capacious review of the mathematical models for many medical-biological processes, based on the well-known models and methods of the mechanics of continuous media is presented in [1]. More deepened analysis of medical-biological and mathematical aspects is proposed, particularly, in [2-6].

Mathematical modeling of the normal physiological and pathological processes is the most topical direction in the scientific study. The reason is that the modern medicine is mainly experimental science with the great empirical practice of influence on the disease course by the different facilities. But experiments are restricted concerning to the detailed study of many processes I n alive organism. The most effective apparatus in this case can serve the mathematical modeling. Development of the apparatus means the construction of closed mechanical-mathematical model of the process describing the behavior of the biological media basing on the partial differential equations. Herewith the necessary elements are such as determination of the rheological relationships, describing the behavior of some media part (equation mode, relationship between the components of the stress and strain tensors, etc.). The corresponding initial and boundary conditions are necessary for the mathematical problem correctness as well.

Numerical modeling of the biomechanical processes in the medical practice is realized using the models of mechanics of continuous media and numerical methods of solving the corresponding partial differential equations systems. Such modeling takes into account the development and realization of the numerical methods, adapted to the specified concrete tasks, development of the numerical method algorithm and its program package, visualization of the obtained results. Examples of the successful use of the mentioned mathematical methods are presented in [7] to solve the problems of the nonlinear dynamics in the chemical kinetics. To study some medical processes there is necessary to solve numerically the differential equations systems [6]. Biological and medical problems involving to numerical solutions of the partial differential equations are described in the papers [5, 6]. Rheological relationships for the biological continuous media are developed in [8, 9]. The mechanical model of the heart was considered in [10-14]. Description of the simplest mathematical models of the circulatory system and heart one can find in [15-17]. Circulatory system consisting from the large and lesser circulation, possesses very serious and different functions, that is why their modeling in the normal and pathological conditions, is very important task in medicine. For today the most adequacy to the real physical circulatory systems there are dynamical models of the pulsating flows of the incompressible fluids in the elastic tubes system.

2 Problem statement. Mathematical models of blood circulation within the theory of multilayer elastic cylindrical tube

2.1 Linear mathematical models of hereditary biofactor tube

Study of the wave propagation process in the deformed tubes with the liquid leaking through the tube is widespread applicated [18]. These problems are actual, in particular, in case of blood circulation modeling in alive organisms. Problems of blood flows and oscillations propagation in large blood vessels are very important to understand the functioning, regulation and control the cardiovascular system. As follows, diagnostics, surgery and prosthetics are bound up the hemodynamics [19, 20]. In the mathematical modeling of blood flow there is considered pulsed systaltic blood flow in the multilayer elastic or viscous elastic tube with the variable cross-section. More complicated mathematical models of blood circulation in the tubes that possess reaction on the external action (biofactor). This type models describe blood circulation in arteries and veins. Proposed mathematical models are obtained using continuum media mechanics and hereditary theory approach [19, 20].

There is given a multilayer cylindrical tube (vessel) with radius $R(x)$, length l (finite of semi-infinite) and depth h. This tube consists of n different by depth layers δ_s ($h = \sum_{s=1}^{n} \delta_s$), which are connected by common circular concentric surfaces between them. Hereby, $R(x)$ is monotonically decreasing (nonincreasing) function. Hydrodynamic pressure on the vessels walls is described by formula

$$P(x,t) = \sum_{s=1}^{n} \delta_s E_s \left[W(x,t) - \int_0^\infty \Gamma_s(\theta) \cdot W(x,t-\theta) d\theta \right],$$

where:

- in case of the linear elasticity law $\sigma_s = e_s E_s$, σ_s is stress, e_s is deformation in the corresponding s – layer;
- in case of the linear hereditary elasticity law

$$\sigma_s = e_s E_s^*,\ E_s^* = E_s(1 - \Gamma_s^*),$$

Γ_s^* is stress relieving operator, $\Gamma_s^* f = \int_0^\infty \Gamma_s(\theta) f(t-\theta) d\theta$;

- $W(x,t)$ is radial displacement of the wall for the multilayer package as a whole, it being known that

$$\sigma_s = E_s \frac{W}{R} \text{ or } \sigma_s = E_s^* \frac{W}{R}$$

in the linear elasticity and the linear heredity cases, respectively.

- the circular intension N is determined by formula $N = \sum_{s=1}^{n} \delta_s \sigma_s$. The next formulae are valid in the linear elasticity and the linear heredity cases, respectively,

$$P(x,t) = \alpha_n \frac{W}{R^2}$$

or

$$P(x,t) = \frac{1}{R^2} \sum_{s=1}^{n} \delta_s E_s \left[W(x,t) - \int_0^\infty \Gamma_s(\theta) \cdot W(x,t-\theta) d\theta \right],$$

α_n is some positive constant, depending on the elastic characteristics of the vessels walls and layers quantity.

More complicated model predicts the reaction of the vessels walls on the external action [19]. This mathematical model is based on the hypothesis that the body (vessel) material possesses a property to react on the external irritants changing its elastic characteristics. According to this hypothesis true stress σ_s in every layer at that time instant equals to the sum of passive stress σ_s^0 and the reaction (biological factor) R_{biol}^*, besides that $\sigma_s = \sigma_s^0 + R_{biol}^*(t)$, where the biofactor depends on the applied intension value in time moment, directly preceding to the given one. Thus, $0 < A_s < 1$, τ is time delay of the reaction, $\tau \ll t$. Assume that $\sigma_s = (1 - \sigma_s)\sigma_s^0 + A_s \tau \frac{\partial \sigma_s^0}{\partial \tau}$. As above, for the linear elasticity

$$\sigma_s^0 = E_s \frac{W}{R},$$

and for the linear heredity

$$\sigma_s^0 = E_s^* \frac{W}{R}.$$

Taking into consideration all mentioned about the vessels walls intensions in this mathematical model, one can accept that

$$P(x,t) = \frac{1}{R^2}\sum_{s=1}^{n} \delta_s E_s \left[(1-A_s)W(x,t) + A_s\tau\frac{\partial W}{\partial \tau}\right]$$

for the linear elasticity case and

$$P(x,t) = \frac{1}{R^2}\sum_{s=1}^{n} \delta_s E_s^{*} \left[(1-A_s)W(x,t) + A_s\tau\frac{\partial W}{\partial \tau}\right]\Big(W(x,t) -$$
$$-0\infty\Gamma s\theta\cdot Wx,t-\theta d\theta \qquad (1)$$

for the linear heredity case.

2.2 Hemodynamics equation in the linear mathematical models of blood circulation

Notice is that from the formula (1) one can get different mathematical models of the deformed solid body, namely:

- *linear elastic tube* model ($A_s = \Gamma_s = 0$),
- *linear hereditary tube* model ($A_s = 0,\ \Gamma_s \neq 0$),
- *linear elastic tube with the reaction (biofactor)* model ($A_s \neq 0,\ \Gamma_s = 0$).

Hemodynamics equation differs in every case.

A. The linear elastic tube [19]. Oscillations equation of the blood flow is the next:

$$\frac{\partial}{\partial x}\left(\frac{1}{R^3}\frac{\partial Q}{\partial x}\right) - \frac{2\rho}{R^2}\left(\frac{\partial^2 Q}{\partial t^2} + \frac{8\chi}{R^2}\frac{\partial Q}{\partial t}\right) = 0,$$

where $Q = SU$is blood consumption, $S(x) = \pi R^2(x)$ is area of the tube cross-section, $U(x,t)$ is blood averaged flow rate, ρ is blood density, χ is coefficient of kinematic viscosity.

B. The linear hereditary tube [19]. Oscillations equation is:

$$\sum_{s=1}^{n} \delta_s E_s \frac{\partial}{\partial x}\left[\frac{1}{R^3}\left(\frac{\partial Q}{\partial x} - \int_0^\infty \Gamma_s(\theta)\cdot\frac{\partial Q(x,t-\theta)}{\partial x}d\theta\right)\right] - \frac{2\rho}{R^2}\left(\frac{\partial^2 Q}{\partial t^2} + \frac{8\chi}{R^2}\frac{\partial Q}{\partial t}\right) = 0.$$

C. The linear hereditary biofactor tube [20]. Oscillations equation is:

$$\sum_{s=1}^{n} \delta_s E_s \frac{\partial}{\partial x}\left[\frac{1}{R^3}\left((1-A_s)\frac{\partial Q}{\partial x} + A_s\tau\frac{\partial^2 Q}{\partial x\partial t}\right)\right] - \frac{2\rho}{R^2}\left(\frac{\partial^2 Q}{\partial t^2} + \frac{8\chi}{R^2}\frac{\partial Q}{\partial t}\right) = 0.$$

Solutions of these equations can be found as finite sum of the main oscillation and the higher harmonics, using the harmonic analysis methods.

2.3 The nonlinear mathematical model of the hereditary biofactor tube

To study complicated impulses, specific to the circulatory system, it is often necessary to consider the nonlinear models instead the linear ones. It is impossible to find the exact solution in this case. Background of the nonlinear mathematical models study is numerical (computer) modeling. Application of this approach also can't be universal study method due to the other problems, for example, procedure convergence, numerical method's stability, accuracy of computation. That's why it is reasonable to develop hybrid methods to study the nonlinear oscillations mathematical models combining both the qualitative and numerical approaches [21, 22]. It is realized thorough qualitative description of the solution's characteristics respectively to the problem. Usually, it is based on Galerkin method or on its different modifications. After that the numerical methods are applicated to find the approximate solutions. Herewith choice of the numerical method is of no principle from the theoretical aspect, and can be determined by effectiveness of the numerical realization only.

Using all mentioned above, let's consider the oscillations equation of blood flow in the nonlinear isotropic environment. It would be studied the problem of the blood flow oscillations equation within the mathematical model of the linear hereditary tube with biofactors and nonlinear external dissipative forces in the form

$$\frac{\partial^2 Q}{\partial t^2} = a^2 \frac{\partial^2 Q}{\partial x^2} + \frac{\zeta}{\rho_l} \frac{\partial^3 Q}{\partial x^2 \partial t} - \frac{\nu}{\rho_l} \left| \frac{\partial Q}{\partial t} \right|^{p-2} \frac{\partial Q}{\partial t}, \tag{2}$$

where $a = \sqrt{\frac{N_l}{\rho_l}}$, and ρ_l is the linear blood density in the finite tube with fixed length l, N_l is uniformly distributed by tube length force, causing the initial oscillations, ν, ζ are the coefficients of the external and internal dissipations of the environment, respectively, $p > 2$. The nonlinear oscillations of the tube with the constant radius of the cross-section R in the case of the initial amplitude replacement and the initial zero rate would be considered. The case of rigidly fixed by length tube also would be considered. It is reasonable to study the equation (2) in the rectangle $\Pi_T = [0; l] \times [0; T]$ mixed problem with the initial conditions

$$Q(x,0) = Q_0(x), \; \frac{\partial Q}{\partial t}(x,0) = 0 \tag{3}$$

and the boundary conditions

$$Q(0,t) = Q(l,t) = 0. \tag{4}$$

For the case $p = 2$ the equation (2) is considered in the previous chapter within the linear elastic tube with the reaction (biofactor) model. In this paper the linear model is involved by the nonlinear factors.

Discussed procedure for the nonlinear mathematical model of blood circulation (2), (3), (4) would be illustrated in the next chapters.

3 Study of the problem solution existence and uniqueness via Galerkin method

Mixed problem (2), (3), (4) is the nonlinear evolutionary third-order equation problem. Hence there are no any analytical methods to find the solutions, then to solve the problem it is necessary to applicate the numerical methods. There is presented the procedure for the qualitative study of the solution in the mathematical model of small transverse nonlinear oscillations of the vessel that allows to obtain the solution existence and uniqueness for the problem (2), (3), (4). After the correctness justification the question of the numerical method choice is principled only on its effectiveness.

The generalized solution of the problem (2), (3), (4) in the domain Π_T will be denominated the function $Q(x,t)$, satisfying the conditions (3), (4) and the integral identity

$$\int_0^\tau \int_0^l \left[-\frac{\partial Q}{\partial t}\frac{\partial V}{\partial t} + \frac{\zeta}{\rho_l}\frac{\partial^2 Q}{\partial x \partial t}\frac{\partial V}{\partial x} + a^2 \frac{\partial Q}{\partial x}\frac{\partial V}{\partial x} + \frac{\nu}{\rho_l}\left|\frac{\partial Q}{\partial t}\right|^{p-2}\frac{\partial Q}{\partial t}V\right] dxdt +$$
$$+ \int_0^l \frac{\partial Q(x,\tau)}{\partial t} V(x,\tau)dx = 0 \qquad (5)$$

for the arbitrary $\tau \in [0;T]$ and for the arbitrary respectively chosen function V. The qualitative properties of the solution are the next:

- the functions Q and $\frac{\partial Q}{\partial t}$ are continuous on the variable t on the section $[0;T]$, the function $\frac{\partial Q}{\partial t}$ is integrable by Lebesgue with power p on $[0;T]$;
- on the variable x the function Q with the second derivative $\frac{\partial^2 Q}{\partial x^2}$ are integrable by Lebesgue with the second degree on $(0;l)$;
- on the variable x the function $\frac{\partial Q}{\partial t}$ is integrable by Lebesgue with power p on $(0;l)$.

The main result of the qualitative study of the problem solution is the next: under the condition $Q_0 \in H_0^1(0;l)$ the generalized unique solution $Q(x,t)$ exists for the problem (2), (3), (4) in Π_T.

Let's introduce the procedure of obtaining the main result. To justify the solution existence of the problem (2), (3), (4) it would be considered in the domain Π_T the sequence of the Gakerkin's approximations $Q^N(x,t) = \sum_{k=1}^N c_k^N(t)\,\omega^k(x)$, $N = 1,2,...,\{\omega^k\}$ is orthonormalized in $L^2(0,l)$ system of the linear independent elements of the space $H_0^1(0,l) \cap L^p(0,l)$, such that the linear combinations $\{\omega^k\}$ are dense in $H_0^1(0,l) \cap L^p(0,l)$. In addition to that the functions c_k^N are determined as Cauchy problem solutions for the ordinary differential equations system

$$\int_0^l \left[\left(\frac{\partial^2 Q^N}{\partial t^2} + \frac{\nu}{\rho_l}\left|\frac{\partial Q^N}{\partial t}\right|^{p-2}\frac{\partial Q^N}{\partial t}\right)\omega^k + \left(\frac{\zeta}{\rho_l}\frac{\partial^2 Q^N}{\partial x \partial t} + a^2\frac{\partial Q^N}{\partial x}\right)\frac{\partial \omega^k}{\partial x}\right] dx = 0, \qquad (6)$$

where $k = 1,2,\dots,N$, with the initial conditions

$$c_k^N(0) = Q_{0,k}^N, \qquad \frac{\partial c_k^N}{\partial t}(0) = 0, \qquad (7)$$

$Q_0^N(x) = \sum_{k=1}^N Q_{0,k}^N(x)\,\omega^k$, $\|Q_0^N - Q_0\|_{H_0^1(0,l)} \to 0$, $N \to \infty$. On the basis of Karatheodori theorem [23, p. 28] there exists an absolutely continuous solution for the problem (6), (7), determined in a certain interval $[0,t_0)$. Due to the evaluations obtained below, it would follow that $t_0 = T$, while number T would be determined later. Multiplying (6) by $\frac{\partial c_k^N}{\partial t}$, summing it up by k from 1 to N and integrating it by t from 0 to $\tau \le T$, one can get

$$\int_0^\tau \left\langle \frac{\partial^2 Q^N}{\partial t^2}, \frac{\partial Q^N}{\partial t} \right\rangle dt +$$

$$+\int_0^\tau \int_0^l \left[\frac{\zeta}{\rho_l} \frac{\partial^2 Q^N}{\partial x \partial t} \frac{\partial^2 Q^N}{\partial x \partial t} + a^2 \frac{\partial Q^N}{\partial x} \frac{\partial^2 Q^N}{\partial x \partial t} + \frac{\nu}{\rho_l} \left| \frac{\partial Q^N}{\partial t} \right|^p \right] dxdt = 0. \qquad (8)$$

Let's realize the transformations and estimations of the integrals in the equality (8). As result it is possible to show that limiting function Q satisfies the integral identity (5), conditions (3), (4), and also possesses corresponding qualitative properties. Thus, Q is the generalized solution of the problem (2), (3), (4). To establish uniqueness let's denote $W = Q^1 - Q^2$, where Q^1, Q^2 are two generalized solutions of the problem (2), (3), (4). Since $Q^1(x,0) = Q^2(x,0)$, $\frac{\partial Q^1(x,0)}{\partial t} = \frac{\partial Q^2(x,0)}{\partial t} = 0$, then

$$\int_0^l \left[\left(\frac{\partial W(x,\tau)}{\partial t} \right)^2 + \left(\frac{\partial W(x,\tau)}{\partial x} \right)^2 \right] dx + \int_0^\tau \int_0^l \left[\left(\frac{\partial^2 W(x,\tau)}{\partial x \partial t} \right)^2 \times \right.$$

$$\left. \times \left(\left| \frac{\partial Q^1}{\partial t} \right|^{p-2} \frac{\partial Q^1}{\partial t} - \left| \frac{\partial Q^2}{\partial t} \right|^{p-2} \frac{\partial Q^2}{\partial t} \right) \left(\frac{\partial Q^1}{\partial t} - \frac{\partial Q^2}{\partial t} \right) \right] dxdt \leq 0$$

Hence, $Q^1 = Q^2$ almost everywhere in Π_T, whence it follows the solution uniqueness.

4 Model case of the elastic tube with the reaction and computer modeling results

For the mathematical modeling of the free nonlinear small transversal oscillations there is used long fixed on the endpoints tube under the force action on the unit of length N_l. Free oscillations of the blood flow are described by the problem (2), (3), (4). Under these conditions the space discretization of the equation (2) is realized. Let n be the quantity of the discretization components, and Δx be difference interval over the coordinate x. Solving of the problem (2), (3), (4) results in the numerical integration on the time interval of some difference equations system under some initial conditions. Computer modeling of the transient processes was realized on the model example of the analysis of the small transversal oscillations of the elastic thin tube $R = 0{,}003\ m$ with the finite length. Tube length is 1 m, blood density $\rho = 1058 \frac{\text{kg}}{m^3}$, wall thickness is h m. Tube is under the initial pertubation of the force applied to its center in the dilation direction (that means perpendicular to the line length). System parameters are the next: $N_l = 50 \frac{N}{m}$, $\Delta x = 0{,}1\ m$, $\zeta = 1\ \frac{\text{kg}\cdot m}{s}$. Integration of the equations in the mechanical mode is realized via the explicit Runge-Kutta fourth order method and implicit second order Geer method. Numerical results almost coinside. Integration step of the explicit Runge-Kutta method is $1 \cdot 10^{-5}\, s$, implicit Geer method is $1 \cdot 10^{-4}\, s$. The nonlinear algebraic equations system on every step by the variable t is solved via the simple iteration method.

Numerical simulations were performed using the Fortran software subroutine package gfortran.

Three modes of the object were studied. The first mode presents the oscillations of the sufficiently small flows in the tube (the linear internal dissipation of the mechanical energy is present only, $\nu = 0$). The second mode presents half-filled tube (the linear internal and linear external dissipations of the mechanical energy are present, $p = 2$, $\nu = 3$). The third mode presents the oscillations of the filled tube (the linear internal and the nonlinear external dissipations of the mechanical energy are present, $p = 4{,}3$, $\nu = 3$). Obviously, that these assumptions are adapted, but even in this case with the sufficient adequacy extent there are described the real physical processes in the object. Thus, there realized three experiments taking into consideration mentioned modes. To confirm the validity of the system model also were carried out two additional experiments consisting in study of the transitional processes with different tube thickness. The first experiment examined $h = 0{,}002\ m$ (Fig. 1a, 1b), the second experiment examined $h = 0{,}001\ m$ (Fig. 2).

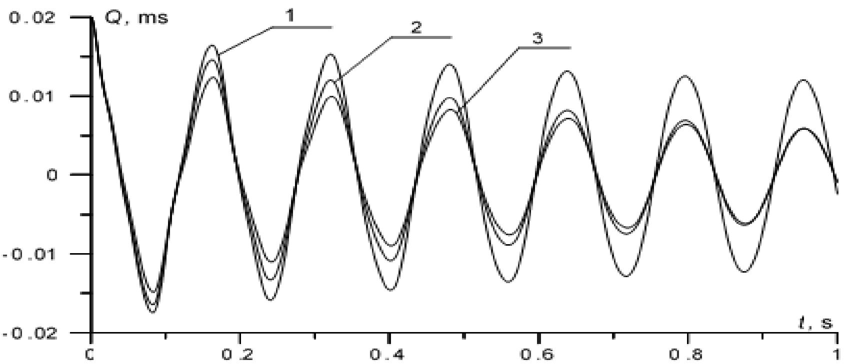

Figure 1a. Transient motions of the tube central component $t \in [0; 1]$, ($h = 0{,}002m$): 1 is the first experiment, 2 is the second experiment, 3 is the third experiment

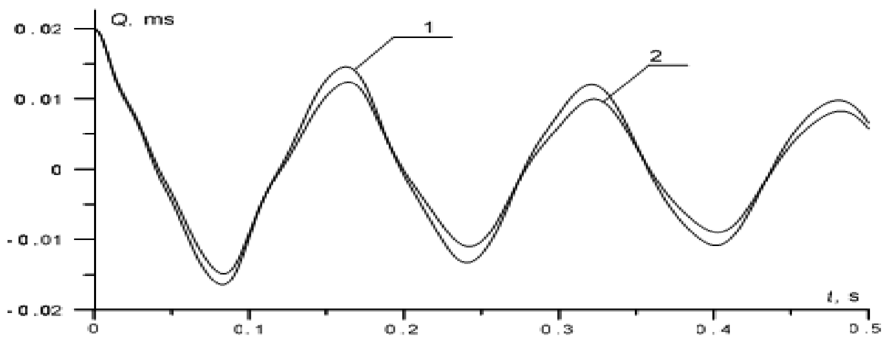

Figure 2b. Transient motions of the tube central component ($h = 0{,}002m$): 1 is the second experiment, 2 is the third experiment at time span $t \in [0; 0{,}5]$

Analyzing the family of curves, one can see the essential influence of the internal dissipative processes on the tube oscillations. Partly filled tube can be treated as an object with the linear external dissipation in contradistinction to the fully filled tube. Turbulent processes of the blood circulation cause the nonlinear influence on the vessel walls. This influence depends on the vessel wall thickness. This fact is clearly fixed on the Fig. 2. Reducing of the tube thickness causes the increasing of the eigenoscillation frequency and contrariwise.

This fact absolutely corresponds to the classical elasticity theory. The oscillations damping in the tube with the less thickness are more intensive being dependent on the internal processes in the vessel body. The nonlinear characteristics of the blood flow are manifested stronger in the vessels with the thin walls, that is well-understood from the physical view of point.

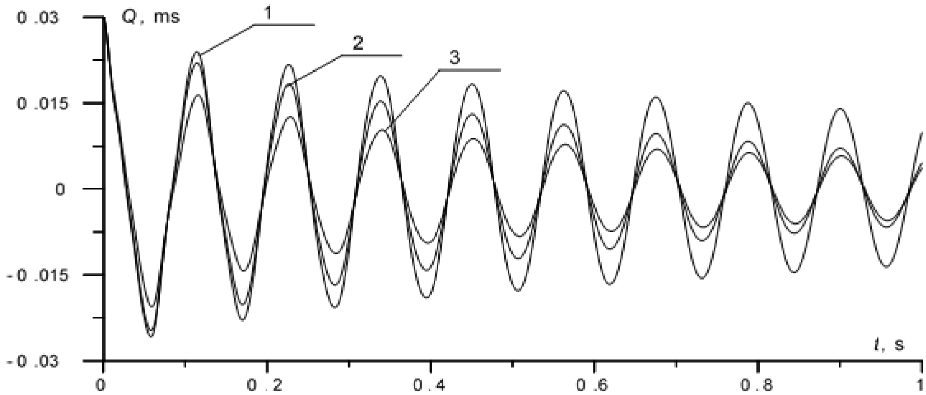

Figure 2: Transient motions of the tube central component ($h = 0{,}001m$): 1 is the first experiment, 2 is the second experiment, 3 is the third experiment

5 Conclusions

The elaboration of the mathematical models of the physiological processes in the able-bodied organism, and also medical problems that follow in the sick mode of the patient, can be considered as mathematical modeling domain that is intensively developed. Hereby the qualitative study of the mathematical model and the next numerical modeling often are effective and available instrument of the biological and medical problems investigation. Complication and detail working out of the physical and mathematical problems in the mathematical modeling directly explained by the high rate development of the numerical resources and also instrumental and diagnostics facilities. This provides the science insight on the principally new levels of the medical and biological processes understanding. There is a lot of papers with the fundamental scientific part in the foreground. In days to come this fundamental part will determine the priority of the computing procedures to satisfy highly effective medical attendance, health and active longevity of the human.

To confirm the problem correctness in the nonlinear mathematical model of the blood circulation in the paper are used the fundamental methods of the nonlinear boundary problems general theory. Basing on the results of the numerical modeling there is proved the sufficient adequacy of the obtained model to the real prototype. It is shown that the nonlinear medium promotes to the quicker oscillations damping and causes the inharmonic processes in the system. The well-known fact, that increasing of the vessel walls thickness causes the reducing eigenoscillations frequency of the system and contrariwise also is reaffirmed.

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ПРО ОДНУ НЕЛІНІЙНУ МАТЕМАТИЧНУ МОДЕЛЬ КРОВООБІГУ З РЕАКЦІЄЮ СТІНОК СУДИН У РАМКАХ СПАДКОВОЇ ТЕОРІЇ

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Дослідження репрезентує достатні умови існування та єдиності розв’язку однієї змішаної задачі, яка використовується в коливальній математичній моделі кровообігу під дією нелінійних дисипативних сил у рамках теорії спадкової трубки з біофактором. Отримані якісні результати обґрунтовують застосування методу Гальоркіна до вищезазначеної задачі. Ці результати сприяли чисельному моделюванню та застосуванню різних (явних і неявних) чисельних методів у подальших дослідженнях динамічних характеристик розв’язків у розглянутих математичних моделях коливань. В рамках цього дослідження чисельне інтегрування рівнянь руху за методом Рунге-Кутта 4-го порядку та методом Гіра 2-го порядку в модельному випадку дозволило оцінити вплив різних фізико-механічних факторів на амплітуду та частоту коливального процесу. Використання гібридних методів для моделювання коливань у нелінійному ізотропному пружному середовищі на прикладі кровообігу у судинах дозволило сформулювати рівняння механічного стану об’єкта на основі енергетичних підходів та теорії механічних полів у континуальних середовищах.

Ключові слова: *математична модель, нелінійні коливання, метод Гальоркіна, біофактор, кровообіг, судина*.