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REMARKS ON SOME CLASSES OF POSITIVE CONTINUOUS FUNCTIONS IN $\mathbb{C}^n$

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Here we prove two propositions providing sufficient conditions of belonging positive continuous functions in $\mathbb{C}^n$ to classes Q^n and $Q^n_{\boldsymbol{b}}$. These auxiliary classes plays important role in theory of entire functions of bounded L-index in direction and bounded $\boldsymbol{L}$-index in joint variables, where $L\colon \mathbb{C}^n \to \mathbb{R}_+, \boldsymbol{L}\colon \mathbb{C}^n \to \mathbb{R}^n_+$ are continuous functions. They help to constuct general theory of bounded index for very wide class of entire functions, because for every entire functions with bounded multiplicities of zero points there exists a corresponding function L or $\boldsymbol{L}$ providing boundedness of L-index in direction or boundedness $\boldsymbol{L}$-index in joint variables respectively. Our result requires uniform boundedness of logarithmic derivative in all variables z_j and $\overline{z}_j$ for belonging the function to class Q^n, $j \in \{1, \ldots, n\}$. Another result requires uniform boundedness of logarithmic derivative in directions $\boldsymbol{b}$ and $\overline{\boldsymbol{b}}$ for belonging the function to class $Q^n_{\boldsymbol{b}}$, where $\overline{\boldsymbol{b}}$ is the complex conjugate vector to $\boldsymbol{b}$.

Keywords: *positive continuous function, several complex variables, partial logarithmic derivative, local behavior, complex conjugate.*

1 Introduction

Let $\mathbf{b} \in \mathbb{C}^n \setminus \{0\}, L\colon \mathbb{C}^n \to \mathbb{R}_+, \mathbf{L}\colon \mathbb{C}^n \to \mathbb{R}^n_+$. The notions of boundedness of L-index in direction and boundedness of $\mathbf{L}$-index in joint variables are very important in theory of holomorphic functions of several complex variables (see more in [1, 2, 3]). The functions from these classes has many interesting properties, describing its local behavior and its asymptotic behavior (as growth estimates). Moreover, they have bounded value distribution in a some sense. To construct flexible theory for these functions mathematicians assume some restrictions by behavior of the functions L and **L**. Typically, L and **L** are positive and continuous. But these restrictions are week. Therefore, there were introduced auxiliary classes $Q_{\mathbf{b}}^n$ and Q^n (see definitions below). In 2015, Prof. S.Yu. Favorov set up a problem *to describe functions from $Q_{\boldsymbol{b}}^n$ by its differential characteristics*. Bandura A. and Skaskiv O. obtained some proposition of such type in [4]. Below we formulate the statement as Proposition 1. A similar proposition for functions Q^n was obtained in [5] (see below Lemma 1).

In 2018, Prof. S.Yu. Favorov noted that proofs of these assertions contain gap. At the fact the proofs concern entire functions L and **L** even formulations of the propositions contain condition that all first order partial derivative of the vector-valued function **L** and first order derivative of the function L in the direction **b** are continuous in $\mathbb{C}^n$. This gap can be removed by two approaches.

At the fist, the condition that 'L and **L** and their first order partial and directional derivative are continuous' can be replaced by the condition that 'the functions L and **L** are entire'. After such replacement the proofs of propositions do not change and their conclusion are valid.

At the second, the condition 'L and **L** and their first order partial and directional derivative are continuous' can be extended by the additional condition on the partial derivatives in variables $\overline{z}_j$ and derivative in the direction $\overline{b}$, where $\overline{b}$ is the complex conjugate vector to **b** and $\overline{z}_j$ is the complex conjugate of z_j, $j \in \{1, \dots, n\}$.

An implementation of the second approach is **a goal of the presented paper**.

2 The class $Q_{\mathbf{b}}^n$

For $\eta > 0, z \in \mathbb{C}^n, \mathbf{b} = (b_1, \dots, b_n) \in \mathbb{C}^n \backslash \{\mathbf{0}\}$and a positive continuous function $L: \mathbb{C}^n \to \mathbb{R}_+$ we define

$$\lambda_1^{\mathbf{b}}(z, \eta, L) = \inf\left\{\frac{L(z + t\mathbf{b})}{L(z)} : |t| \leq \frac{\eta}{L(z)}\right\},$$
$$\lambda_1^{\mathbf{b}}(\eta, L) = \inf\{\lambda_1^{\mathbf{b}}(z, \eta) : z \in \mathbb{C}^n\},$$
$$\lambda_2^{\mathbf{b}}(z, \eta, L) = \sup\left\{\frac{L(z + t\mathbf{b})}{L(z)} : |t| \leq \frac{\eta}{L(z)}\right\},$$
$$\lambda_2^{\mathbf{b}}(\eta, L) = \sup\{\lambda_2^{\mathbf{b}}(z, \eta, L) : z \in \mathbb{C}^n\}.$$

By $Q_{\mathbf{b}}^n$ we denote the class of functions L which satisfy the condition

$$(\forall \eta \geq 0): 0 < \lambda_1^{\mathbf{b}}(\eta, L) \leq \lambda_2^{\mathbf{b}}(\eta, L) < +\infty. \tag{1}$$

Let us denote $\frac{\partial L}{\partial \mathbf{b}} = \sum_{j=1}^n \frac{\partial L}{\partial z_j} b_j$. It was proved such a proposition on sufficient conditions of belonging to class $Q_{\mathbf{b}}^n$ in terms of differential characteristics of functions.

Proposition 1 (index-cos-sqrt) *Let $G \subset \mathbb{C}^n, L: G \to \mathbb{C}$ and $\frac{\partial L}{\partial \boldsymbol{b}}$ be continuous functions in a domain G. If there exist numbers $P > 0$ and $c > 0$ such that for all $z \in \overline{G}$*

$$\frac{1}{c + |L(z)|}\left|\frac{\partial L(z)}{\partial \mathbf{b}}\right| \leq P$$

then for every $\eta \geq 0$ inequalities

$$0 < \inf_{z\in\overline{G}} \inf_{\substack{t_0\in\mathbb{C},\\ z+t_0\mathbf{b}\in\overline{G}}} \inf_{|t-t_0|\leq\frac{\eta}{L_1(z+t_0\mathbf{b})}} \frac{L_1(z+t\mathbf{b})}{L_1(z+t_0\mathbf{b})} \leq$$

$$\leq \sup_{z\in\overline{G}} \sup_{\substack{t_0\in\mathbb{C},\\ z+t_0\mathbf{b}\in\overline{G}}} \sup_{|t-t_0|\leq\frac{\eta}{L_1(z+t_0\mathbf{b})}} \frac{L_1(z+t\mathbf{b})}{L_1(z+t_0\mathbf{b})} < \infty$$

holds, where $L_1(z) = c + |L(z)|$. If, in addition, $G = \mathbb{C}^n$ then $L_1 \in Q_{\mathbf{b}}^n$.

Every complex function $L: \mathbb{C}^n \to \mathbb{C}$ can be represented as $L(z) = u(x,y) + iv(x,y)$ with $x \in \mathbb{R}^n, y \in \mathbb{R}^n, z = x + iy, v: \mathbb{R}^{2n} \to \mathbb{R}, u: \mathbb{R}^{2n} \to \mathbb{R}$. It is known that $\frac{\partial L}{\partial z_j} = \frac{1}{2}(\frac{\partial L}{\partial x} - i\frac{\partial L}{\partial y})$ Traditionally,

$$\frac{\partial L}{\partial \overline{z}_j} := \frac{1}{2}\left(\frac{\partial L}{\partial x} + i\frac{\partial L}{\partial y}\right).$$

Therefore, we define

$$\frac{\partial L}{\partial \overline{\mathbf{b}}} := \sum_{j=1}^{n} \frac{\partial L}{\partial \overline{z}_j}\overline{b}_j,$$

where $\overline{b_j}$ is a complex conjugate of b_j, i.e. for $b_j = x_j + iy_j$ one has $\overline{b_j} = x_j - iy_j, (x_j, y_j \in \mathbb{R})$.

Proposition 2 *Let $L: \mathbb{C}^n \to \mathbb{C}$ and $\frac{\partial L}{\partial \boldsymbol{b}}, \frac{\partial L}{\partial \overline{\boldsymbol{b}}}$ be continuous functions. If there exist numbers $P > 0$ and $c > 0$ such that for all $z \in \mathbb{C}^n$*

$$\frac{1}{c + |L(z)|}\left|\frac{\partial L(z)}{\partial \mathbf{b}}\right| \leq P,$$

$$\frac{1}{c + |L(z)|}\left|\frac{\partial L(z)}{\partial \overline{\mathbf{b}}}\right| \leq P$$

then for every $\eta \geq 0$ inequalities

$$0 < \inf_{z\in\mathbb{C}^n} \inf_{|t|\leq\frac{\eta}{L_1(z)}} \frac{L_1(z+t\mathbf{b})}{L_1(z)} \leq \sup_{z\in\mathbb{C}^n} \sup_{|t|\leq\frac{\eta}{L_1(z)}} \frac{L_1(z+t\mathbf{b})}{L_1(z)} < \infty$$

hold, where $L_1(z) = c + |L(z)|, c \in \mathbb{R}_+$. And $L_1 \in Q_{\mathbf{b}}^n$.

Proof of Proposition 2. Clearly, the function $L_1(z)$ is positive and continuous. For a given $t \in \mathbb{C}$ we define an analytic curve $\varphi(\tau) = \tau e^{i\arg(t)}, \tau \in [0, |t|]$. For every continuously differentiable function g of real variable τ the inequality $\frac{d}{dt}|g(\tau)| \leq |g'(\tau)|$ holds except the points where $g'(\tau) = 0$. Using restrictions of this proposition, we deduce the upper estimate of $\lambda_2^{\mathbf{b}}(z, \eta)$ for the function L_1:

$$\lambda_2^{\mathbf{b}}(z,\eta,L_1)=\sup\left\{\frac{L_1(z+t\mathbf{b})}{L_1(z)}:|t|\le\frac{\eta}{L_1(z)}\right\}=$$
$$=\sup\left\{\frac{c+|L(z+t\mathbf{b})|}{c+|L(z)|}:|t|\le\frac{\eta}{c+|L(z)|}\right\}=$$
$$=\sup_{|t|\le\eta/(c+|L(z)|)}\left\{\exp\{\ln(c+|L(z+t\mathbf{b})|)-\ln(c+|L(z)|)\}\right\}=$$
$$=\sup\left\{\exp\left\{\int_0^{|t|}\frac{d(c+|L(z+\varphi(\tau)\mathbf{b})|)}{c+|L(z+\varphi(\tau)\mathbf{b})|}\right\}:|t|\le\frac{\eta}{c+|L(z)|}\right\}\le$$
$$\le\sup_{|t|\le\frac{\eta}{c+|L(z)|}}\left\{\exp\left\{\int_0^{|t|}\frac{|\varphi'(\tau)|\cdot|\mathbf{b}|}{c+|L(z+\varphi(\tau)\mathbf{b})|}\left(\left|\frac{\partial L(z+\varphi(\tau)\mathbf{b})}{\partial\mathbf{b}}\right|\right.\right.\right.$$
$$\left.\left.\left.+\left|\frac{\partial L(z+\varphi(\tau)\mathbf{b})}{\partial\overline{\mathbf{b}}}\right|\right)|d\tau|\right\}\right\}\le$$
$$\le\sup_{|t|\le\frac{\eta}{c+|L(z)|}}\left\{\exp\{2P|\mathbf{b}|\cdot|t|\}\right\}=\exp\left(\frac{2P|\mathbf{b}|\eta}{c+|L(z)|}\right)\le\exp\left(\frac{2P|\mathbf{b}|\eta}{c}\right).$$

Hence, for all $\eta\ge 0\lambda_2^{\mathbf{b}}(\eta,L_1)=\sup_{z\in\mathbb{C}^n}\lambda_2^{\mathbf{b}}(z,\eta,L_1)\le\exp\left(\frac{P|\mathbf{b}|\eta}{c}\right)<\infty$.
Using $\frac{d}{dt}|g(t)|\ge-|g'(t)|$ it can be proved that for every $\eta\ge 0$ one has

$$\lambda_1^{\mathbf{b}}(z,\eta,L_1)=\inf\left\{\frac{c+|L(z+t\mathbf{b})|}{c+|L(z)|}:|t|\le\frac{\eta}{c+|L(z)|}\right\}=$$
$$=\inf_{|t|\le\eta/(c+|L(z)|)}\left\{\exp\{\ln(c+|L(z+t\mathbf{b})|)-\ln(c+|L(z)|)\}\right\}=$$
$$=\inf\left\{\exp\left\{\int_0^{|t|}\frac{d(c+|L(z+\varphi(\tau)\mathbf{b})|)}{c+|L(z+\varphi(\tau)\mathbf{b})|}\right\}:|t|\le\frac{\eta}{c+|L(z)|}\right\}\ge$$
$$\ge\inf_{|t|\le\frac{\eta}{c+|L(z)|}}\left\{\exp\left\{-\int_0^{|t|}\frac{|\varphi'(\tau)|\cdot|\mathbf{b}|}{c+|L(z+\varphi(\tau)\mathbf{b})|}\left(\left|\frac{\partial L(z+\varphi(\tau)\mathbf{b})}{\partial\mathbf{b}}\right|\right.\right.\right.$$
$$\left.\left.\left.+\left|\frac{\partial L(z+\varphi(\tau)\mathbf{b})}{\partial\overline{\mathbf{b}}}\right|\right)|d\tau|\right\}\right\}\ge$$
$$\ge\inf_{|t|\le\frac{\eta}{c+|L(z)|}}\left\{\exp\{-2P|\mathbf{b}|\cdot|t|\}\right\}=\exp\left(\frac{-2P|\mathbf{b}|\eta}{c+|L(z)|}\right)\ge\exp\left(\frac{-2P|\mathbf{b}|\eta}{c}\right).$$

i.e. $\lambda_1^{\mathbf{b}}(\eta,L_1)\ge\exp\left(-\frac{2P|\mathbf{b}|\eta}{c}\right)>0$. Therefore, $L_1\in Q_{\mathbf{b}}^n$.

3 The class Q^n

Denote $\mathbf{0}=(0,\dots,0)\in\mathbb{R}^n$. For $A=(a_1,\dots,a_n)\in\mathbb{C}^n, B=(b_1,\dots,b_n)\in\mathbb{R}^n$, a notation $A<B$ means that $a_j<b_j$ for all $j\in\{1,\dots,n\}$; similarly, the relation $A\le B$ is defined. The closed polydisc $\{z\in\mathbb{C}^n:|z_j-z_j^0|\le r_j,\ j\in\{1,\dots,n\}\}$ is denoted by $D^n[z^0,R]$. For $R\in\mathbb{R}_+^n, j\in\{1,\dots,n\}$ and $\mathbf{L}(z)=(l_1(z),\dots,l_n(z))$ we define

$$\lambda_{1,j}(z_0,R,\mathbf{L})=\inf\left\{l_j(z)/l_j(z^0):z\in D^n[z^0,R/\mathbf{L}(z^0)]\right\},$$

$$\lambda_{2,j}(z_0,R,\mathbf{L})=\sup\{l_j(z)/l_j(z^0): z\in D^n[z^0,R/\mathbf{L}(z^0)]\},$$
$$\lambda_{1,j}(R,\mathbf{L})=\inf_{z^0\in\mathbb{C}^n}\lambda_{1,j}(z_0,R,\mathbf{L}), \lambda_{2,j}(R,\mathbf{L})=\sup_{z^0\in\mathbb{C}^n}\lambda_{2,j}(z_0,R,\mathbf{L}),$$
$$\Lambda_k(R,\mathbf{L})=(\lambda_{k,1}(R,\mathbf{L}),\dots,\lambda_{k,n}(R,\mathbf{L}))(k\in\{1,2\}).$$

By Q^n we denote a class of functions $\mathbf{L}(z)$ which for every $R\in\mathbb{R}_+^n$ satisfy the condition

$$\mathbf{0}<\Lambda_1(R,\mathbf{L})\le\Lambda_2(R,\mathbf{L})<+\infty. \quad (2)$$

It was proved such a proposition on sufficient conditions of belonging to class Q^n in terms of differential characteristics of functions.

Lemma 1 (joint-entire) *Let $\boldsymbol{L}(z)=(l_1(z),\dots,l_n(z)), l_j:\mathbb{C}^n\to\mathbb{C}$ and $\frac{\partial l_j}{\partial z_m}$ be continuous functions in $\mathbb{C}^n$ for all $j,m\in\{1,2,\dots,n\}$. If there exist numbers $P>0$ and $c>0$ such that for all $z\in\mathbb{C}^n$ and every $j,m\in\{1,2,\dots,n\}$*

$$\frac{1}{c+|l_j(z)|}\left|\frac{\partial l_j(z)}{\partial z_m}\right|\le P \quad (3)$$

then $\mathbf{L}^*\in Q^n$, where $\mathbf{L}^*(z)=(c+|l_1(z)|,\dots,c+|l_n(z)|)$.

Below we presented a corrected version of the proposition with conditions by $\overline{z}$.

Proposition 3 *Let $\boldsymbol{L}(z)=(l_1(z),\dots,l_n(z)), l_j:\mathbb{C}^n\to\mathbb{C}$ and $\frac{\partial l_j}{\partial z_m},\frac{\partial l_j}{\partial \overline{z}_m}$, be continuous functions in $\mathbb{C}^n$ for all $j,m\in\{1,2,\dots,n\}$. If there exist numbers $P>0$ and $c_j>0$ such that for all $z\in\mathbb{C}^n$ and every $j,m\in\{1,2,\dots,n\}$*

$$\frac{1}{c_j+|l_j(z)|}\left|\frac{\partial l_j(z)}{\partial z_m}\right|\le P,$$
$$\frac{1}{c_j+|l_j(z)|}\left|\frac{\partial l_j(z)}{\partial \overline{z}_m}\right|\le P,$$

then $\mathbf{L}^*\in Q^n$, where $\mathbf{L}^*(z)=(c_1+|l_1(z)|,\dots,c_n+|l_n(z)|)$.

Proof. Clearly, the function $\mathbf{L}^*(z)$ is positive and continuous. For given $z\in\mathbb{C}^n, z^0\in\mathbb{C}^n$ we define an analytic curve $\varphi:[0,1]\to\mathbb{C}^n$

$$\varphi_j(\tau)=z_j^0+\tau(z_j-z_j^0),\qquad j\in\{1,2,\dots,n\},$$

where $\tau\in[0,1]$. It is known that for every continuously differentiable function g of real variable τ the inequality $\frac{d}{dt}|g(\tau)|\le|g'(\tau)|$ holds except the points where $g'(\tau)=0$. Using restrictions of this proposition, we establish the upper estimate of $\lambda_{2,j}(z_0,R,\mathbf{L}^*)$:

$$\lambda_{2,j}(z_0,R,\mathbf{L}^*)=\sup\left\{\frac{c_j+|l_j(z)|}{c_j+|l_j(z^0)|}: z\in D^n\left[z^0,\frac{R}{\mathbf{L}^*(z^0)}\right]\right\}=$$
$$=\sup_{z\in D^n\left[z^0,\frac{R}{\mathbf{L}^*(z^0)}\right]}\left\{\exp\{\ln(c_j+|l_j(z)|)-\ln(c_j+|l_j(z^0)|)\}\right\}=$$
$$=\sup\left\{\exp\left\{\int_0^1\frac{d(c_j+|l_j(\varphi(\tau))|)}{c_j+|l_j(\varphi(\tau))|}\right\}: z\in D^n\left[z^0,\frac{R}{\mathbf{L}^*(z^0)}\right]\right\}\le$$

$$\leq \sup_{z\in D^n\left[z^0,\frac{R}{\mathbf{L}^*(z^0)}\right]}\left\{\exp\left\{\int_0^1 \sum_{m=1}^n \frac{|\varphi_{m'}(\tau)|}{c_j+|l_j(\varphi(\tau))|}\left(\left|\frac{\partial l_j(\varphi(\tau))}{\partial z_m}\right| + \left|\frac{\partial l_j(\varphi(\tau))}{\partial \overline{z_m}}\right|\right)d\tau\right\}\right\} \leq$$

$$\leq \sup_{z\in D^n\left[z^0,\frac{R}{\mathbf{L}^*(z^0)}\right]}\left\{\exp\left\{\int_0^1 \sum_{m=1}^n 2P|z_m-z_m^0|d\tau\right\}\right\} \leq$$

$$\leq \sup_{z\in D^n\left[z^0,\frac{R}{\mathbf{L}^*(z^0)}\right]}\left\{\exp\left\{\sum_{m=1}^n \frac{2Pr_m}{c_m+|l_m(z^0)|}\right\}\right\} \leq \exp\left(2P\sum_{m=1}^n \frac{r_m}{c_m}\right).$$

Hence, for all $R\geq \mathbf{0}$ $\lambda_{2,j}(R,\mathbf{L}^*) = \sup_{z^0\in\mathbb{C}^n}\lambda_{2,j}(z^0,R,\mathbf{L}^*) \leq \exp\left(2P\sum_{m=1}^n \frac{r_m}{c_m}\right) < \infty$. Using $\frac{d}{dt}|g(t)| \geq -|g'(t)|$ it can be proved that for every $R\geq\mathbf{0}$ one has

$$\lambda_{1,j}(z_0,R,\mathbf{L}^*) = \inf\left\{\frac{c_j+|l_j(z)|}{c_j+|l_j(z^0)|} : z\in D^n\left[z^0,\frac{R}{\mathbf{L}^*(z^0)}\right]\right\} =$$

$$= \inf_{z\in D^n\left[z^0,\frac{R}{\mathbf{L}^*(z^0)}\right]}\left\{\exp\{\ln(c_j+|l_j(z)|)-\ln(c_j+|l_j(z^0)|)\}\right\} =$$

$$= \inf\left\{\exp\left\{\int_0^1 \frac{d(c_j+|l_j(\varphi(\tau))|)}{c_j+|l_j(\varphi(\tau))|}\right\} : z\in D^n\left[z^0,\frac{R}{\mathbf{L}^*(z^0)}\right]\right\} \geq$$

$$\geq \inf_{z\in D^n\left[z^0,\frac{R}{\mathbf{L}^*(z^0)}\right]}\left\{\exp\left\{\int_0^1 -\sum_{m=1}^n \frac{|\varphi_{m'}(\tau)|}{c_j+|l_j(\varphi(\tau))|}\left(\left|\frac{\partial l_j(\varphi(\tau))}{\partial z_m}\right| + \left|\frac{\partial l_j(\varphi(\tau))}{\partial \overline{z_m}}\right|\right)d\tau\right\}\right\} \geq$$

$$\geq \inf_{z\in D^n\left[z^0,\frac{R}{\mathbf{L}^*(z^0)}\right]}\left\{\exp\left\{-\int_0^1 \sum_{m=1}^n 2P|z_m-z_m^0|d\tau\right\}\right\} \geq$$

$$\geq \inf_{z\in D^n\left[z^0,\frac{R}{\mathbf{L}^*(z^0)}\right]}\left\{\exp\left\{-\sum_{m=1}^n \frac{2Pr_m}{c_m+|l_m(z^0)|}\right\}\right\} \geq \exp\left(-2P\sum_{m=1}^n \frac{r_m}{c_m}\right).$$

i.e. $\lambda_{1,j}(R,\mathbf{L}^*) \geq \exp\left(-2P\sum_{m=1}^n \frac{r_m}{c_m}\right) > 0$. Therefore, $\mathbf{L}^*\in Q^n$.

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ЗАУВАЖЕННЯ ПРО ДЕЯКІ КЛАСИ ДОДАТНИХ НЕПЕРЕРВНИХ ФУНКЦІЙ В $\mathbb{C}^n$

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У цій статті доведено два твердження, що містять достатні умови належності додатних неперервних функцій в $\mathbb{C}^n$ до класів Q^n та $Q^n_{\boldsymbol{b}}$. Ці допоміжні класи відіграють важливу роль у теорії цілих функцій обмеженого L-індексу за напрямком та обмеженого $\boldsymbol{L}$-індексу за сукупністю змінних, де $L: \mathbb{C}^n \to \mathbb{R}_+$, $\boldsymbol{L}: \mathbb{C}^n \to \mathbb{R}^n_+$ неперервні функції. Вони допомагають побудувати загальну теорію обмеженого індексу для досить широкого класу цілих функцій, адже для кожної цілої функції існує відповідна функція L або $\boldsymbol{L}$, завдяки яким відповідна ціла функція має обмежений L-індекс за напрямком або обмежений $\boldsymbol{L}$-індекс за сукупністю змінних відповідно. Наш результат вимагає рівномірної обмеженості логарифмічних похідних за всіма змінними z_j та $\overline{z}_j$ для належності функції до класу Q^n, $j \in \{1, \dots, n\}$. Інший результат вимагає рівномірної обмеженості логарифмічної похідної за напрямками $\boldsymbol{b}$ та $\overline{\boldsymbol{b}}$ для належності функції до класу $Q^n_{\boldsymbol{b}}$, де $\overline{\boldsymbol{b}}$ комплексно спряжений вектор до вектора $\boldsymbol{b} \in \mathbb{C}^n \setminus \{\boldsymbol{0}\}$.

Ключові слова: *додатна неперервна функція, декілька комплексних змінних, частинна логарифмічна похідна, локальне поводження, комплексне спряження.*